PROBLEMS CHAPTER 7

Plane Stress

7.2-1 The stresses on the bottom surface of a fuel tanker (figure part a) are known to be $\sigma_x = 7750$ psi, $\sigma_y = 1175$ psi, and $\tau_{xy} = 940$ psi (figure part b).

Determine the stresses acting on an element oriented at an angle $\theta = 55^\circ$ from the $x$ axis, where the angle $\theta$ is positive when counterclockwise. Show these stresses on a sketch of an element oriented at the angle $\theta$.

![Image of fuel tanker](a)

7.2-1 ((a) Can Stock Photo Inc./Johan H)

7.2-2 Solve the preceding problem for an element in plane stress on the bottom surface of a fuel tanker (figure part a); stresses are $\sigma_x = 105$ MPa, $\sigma_y = 75$ MPa, and $\tau_{xy} = 25$ MPa.

Determine the stresses acting on an element oriented at an angle $\theta = 40^\circ$ from the $x$ axis, where the angle $\theta$ is positive when counterclockwise. Show these stresses on a sketch of an element oriented at the angle $\theta$.

![Cross Section and Side View](a)

7.2-2 (a) Can Stock Photo Inc./corepics; (b) Can Stock Photo Inc./scanrail)

7.2-3 The stresses acting on element $A$ on the web of a train rail (see figure part a) are found to be 6500 psi tension in the horizontal direction and 18,500 psi compression in the vertical direction (see figure part b). Also, shear stresses with a magnitude of 3800 psi act in the directions shown.

Determine the stresses acting on an element oriented at a counterclockwise angle of $30^\circ$ from the horizontal. Show these stresses on a sketch of an element oriented at this angle.

![Cross Section](a)

7.2-3 ((a) Can Stock Photo Inc./corepics; (b) Can Stock Photo Inc./scanrail)

7.2-4 Solve the preceding problem if the stresses acting on element $A$ on the web of a train rail (see figure part a of Prob. 7.2-3) are found to be 40 MPa in tension in the horizontal direction and 160 MPa in compression in the vertical direction. Also, shear stresses of magnitude 54 MPa act in the directions shown in the figure.

Determine the stresses acting on an element oriented at a counterclockwise angle of $52^\circ$ from the horizontal. Show these stresses on a sketch of an element oriented at this angle.

![Cross Section](a)
Determine the stresses acting on an element oriented at a clockwise angle of 40° from the horizontal. Show these stresses on a sketch of an element oriented at this angle.

PROB. 7.2-6 (a) Daboost/Shutterstock

7.2.7 The stresses acting on element B (see figure part a) on the web of a wide-flange beam are found to be 14,500 psi in compression in the horizontal direction and 2530 psi in compression in the vertical direction (see figure part b). Also, shear stresses with a magnitude of 3500 psi act in the directions shown.

Determine the stresses acting on an element oriented at a counterclockwise angle of 38° from the horizontal. Show these stresses on a sketch of an element oriented at this angle.

PROB. 7.2-7 (a) Can Stock Photo Inc./Jrekemp

7.2.5 The stresses acting on element B on the web of a train rail (see figure part a of Prob. 7.2-3) are found to be 5700 psi in compression in the horizontal direction and 2300 psi in compression in the vertical direction (see figure). Also, shear stresses of magnitude 2500 psi act in the directions shown.

Determine the stresses acting on an element oriented at a counterclockwise angle of 50° from the horizontal. Show these stresses on a sketch of an element oriented at this angle.

PR. 5.7.2.5

An element in plane stress on the fuselage of an airplane (figure part a) is subjected to compressive stresses with a magnitude of 42 MPa in the horizontal direction and tensile stresses with a magnitude of 9.5 MPa in the vertical direction (see figure part b). Also, shear stresses with a magnitude of 15.5 MPa act in the directions shown.
7.2-2 Solve the preceding problem if the normal and shear stresses acting on element B are 56 MPa, 17 MPa, and 27 MPa (in the directions shown in the figure) and the angle is 40° (clockwise).

PROB 7.2-8

7.2-9 The polyethylene liner of a settling pond is subjected to stresses \( \sigma_x = 350 \) psi, \( \sigma_y = 112 \) psi, and \( \tau_{xy} = -120 \) psi, as shown by the plane-stress element in the figure part a.

Determine the normal and shear stresses acting on a seam oriented at an angle of 30° to the element, as shown in the figure part b. Show these stresses on a sketch of an element having its sides parallel and perpendicular to the seam.

PROB 7.2-9

7.2-10 Solve the preceding problem if the normal and shear stresses acting on the element are \( \sigma_x = 2100 \) kPa, \( \sigma_y = 300 \) kPa, and \( \tau_{xy} = -560 \) kPa, and the seam is oriented at an angle of 22.5° to the element.

PROB 7.2-10

7.2-11 A rectangular plate of dimensions 3.0 in. \( \times \) 5.0 in. is formed by welding two triangular plates (see figure). The plate is subjected to a tensile stress of 500 psi in the longer direction and a compressive stress of 350 psi in the shorter direction.

Determine the normal stress \( \sigma_n \) acting perpendicular to the line of the weld and the shear stress \( \tau_{n} \) acting parallel to the weld. (Assume that the normal stress \( \sigma_n \) is positive when it acts in tension against the weld and the shear stress \( \tau_{n} \) is positive when it acts counterclockwise against the weld.)

PROB 7.2-11

7.2-12 Solve the preceding problem for a plate of dimensions 100 mm \( \times \) 250 mm subjected to a compressive stress of 2.5 MPa in the longer direction and a tensile stress of 12.0 MPa in the shorter direction (see figure).

PROB 7.2-12
7.2.13 At a point on the surface of an elliptical exercise machine the material is in biaxial stress with \( \sigma_x = 1400 \text{ psi} \) and \( \sigma_y = -900 \text{ psi} \), as shown in the figure part a. The figure part b shows an inclined plane \( aa \) cut through the same point in the material but oriented at an angle \( \theta \).

Determine the value of the angle \( \theta \) between zero and 90° such that no normal stress acts on plane \( aa \). Sketch a stress element having plane \( aa \) as one of its sides and show all stresses acting on the element.

7.2.15 An element in plane stress from the frame of a racing car is oriented at a known angle \( \theta \) (see figure). On this inclined element, the normal and shear stresses have the magnitudes and directions shown in the figure.

Determine the normal and shear stresses acting on an element whose sides are parallel to the \( xy \) axes, that is, determine \( \sigma_x, \sigma_y, \) and \( \tau_{xy} \). Show the results on a sketch of an element oriented at \( \theta = 0° \).

7.2.16 Solve the preceding problem for the element shown in the figure.

7.2.17 A gusset plate on a truss bridge is in plane stress with normal stresses \( \sigma_x \) and \( \sigma_y \) and shear stress \( \tau_{xy} \), as shown in the figure. At counterclockwise angles \( \theta = 32° \) and \( \theta = 78° \) from the \( x \) axis, the normal stress is 4200 psi in tension.

If the stress \( \sigma_x \) equals 2650 psi in tension, what are the stresses \( \sigma_y \) and \( \tau_{xy} \)?
7.2.18 The surface of an airplane wing is subjected to plane stress with normal stresses \( \sigma_x \) and \( \sigma_y \) and shear stress \( \tau_{xy} \), as shown in the figure. At a counterclockwise angle \( \theta = 32^\circ \) from the \( x \) axis, the normal stress is 29 MPa in tension, and at an angle \( \theta = 46^\circ \), it is 17 MPa in compression.

If the stress \( \sigma_x \) equals 105 MPa in tension, what are the stresses \( \sigma_y \) and \( \tau_{xy} \)?

\[
\begin{align*}
\sigma_x &= 105 \text{ MPa} \\
\sigma_y &= \ldots \\
\tau_{xy} &= \ldots
\end{align*}
\]

PROB. 7.2.18 (Dabboost/Shutterstock)

7.2.19 At a point on the web of a girder on an overhead bridge crane in a manufacturing facility, the stresses are known to be \( \sigma_x = -4300 \text{ psi} \), \( \sigma_y = 1700 \text{ psi} \), and \( \tau_{xy} = 3100 \text{ psi} \) (the sign convention for these stresses is shown in Fig. 7-1). A stress element located at the same point in the structure (but oriented at a counterclockwise angle \( \theta \) with respect to the \( x \) axis) is subjected to the stresses shown in the figure (\( \sigma_x \), \( \tau_y \), and 2100 psi).

Assuming that the angle \( \theta \) is between zero and 90\(^\circ\), calculate the normal stress \( \sigma_x \), the shear stress \( \tau_y \), and the angle \( \theta \).

\[
\begin{align*}
\sigma_x &= \ldots \\
\sigma_y &= \ldots \\
\tau_{xy} &= \ldots
\end{align*}
\]

PROB. 7.2.19 (© Paul Rapson/Alamy)

**Principal Stresses and Maximum Shear Stresses**

When solving the problems for Section 7.3, consider only the in-plane stresses (the stresses in the \( xy \) plane).

7.3.1 An element in plane stress is subjected to stresses \( \sigma_x = 5750 \text{ psi} \), \( \sigma_y = 1100 \text{ psi} \), and \( \tau_{xy} = 750 \text{ psi} \) (see the figure for Prob. 7.2.1).

Determine the principal stresses and show them on a sketch of a properly oriented element.

7.3.2 An element in plane stress is subjected to stresses \( \sigma_x = 105 \text{ MPa} \), \( \sigma_y = 75 \text{ MPa} \), and \( \tau_{xy} = 25 \text{ MPa} \) (see the figure for Prob. 7.2.2).

Determine the principal stresses and show them on a sketch of a properly oriented element.

7.3.3 An element in plane stress is subjected to stresses \( \sigma_x = -5500 \text{ psi} \), \( \sigma_y = -2000 \text{ psi} \), and \( \tau_{xy} = 1900 \text{ psi} \) (see the figure for Prob. 7.2.3).

Determine the principal stresses and show them on a sketch of a properly oriented element.

7.3.4 The stresses acting on element \( A \) in the web of a train rail are found to be 40 MPa tension in the horizontal direction and 160 MPa compression in the vertical direction (see figure). Also, shear stresses of magnitude 54 MPa act in the directions shown (see the figure for Prob. 7.2.4).

Determine the principal stresses and show them on a sketch of a properly oriented element.

7.3.5 The normal and shear stresses acting on element \( A \) are 6500 psi, 17,300 psi, and 2900 psi (see the figure for Prob. 7.2.4).

Determine the maximum shear stresses and associated normal stresses and show them on a sketch of a properly oriented element.

7.3.6 An element in plane stress from the fuselage of an airplane is subjected to compressive stresses of magnitude 35 MPa in the horizontal direction and tensile stresses of magnitude 6.5 MPa in the vertical direction. Also, shear stresses of magnitude 12.5 MPa act in the directions shown (see the figure for Prob. 7.2.5).

Determine the maximum shear stresses and associated normal stresses and show them on a sketch of a properly oriented element.

7.3.7 The stresses acting on element \( B \) in the web of a wide-flange beam are found to be 14,000 psi compression in the horizontal direction and 2600 psi compression in the vertical direction. Also, shear stresses of magnitude 3800 psi act in the directions shown (see the figure for Prob. 7.2.7).

Determine the maximum shear stresses and associated normal stresses and show them on a sketch of a properly oriented element.
3.5 The normal and shear stresses acting on element B are \( \sigma_x = -46 \text{ MPa}, \sigma_y = -13 \text{ MPa}, \) and \( \tau_{xy} = 21 \text{ MPa} \) (see figure for Prob. 7.2-8).

Determine the maximum shear stresses and associated normal stresses and show them on a sketch of a properly oriented element.

7.3-9 A shear wall in a reinforced concrete building is subjected to a vertical uniform load of intensity \( q \) and a horizontal force \( H \), as shown in the first part of the figure. (The force \( H \) represents the effects of wind and earthquake loads.) As a consequence of these loads, the stresses at point \( A \) on the surface of the wall have the values shown in the second part of the figure (compressive stress equal to 1100 psi and shear stress equal to 480 psi).

(a) Determine the principal stresses and show them on a sketch of a properly oriented element.

(b) Determine the maximum shear stresses and associated normal stresses and show them on a sketch of a properly oriented element.

7.3-11 The stresses at a point along a beam supporting a sign (see figure) are \( \sigma_x = 2250 \text{ psi}, \sigma_y = 1175 \text{ psi}, \) and \( \tau_{xy} = -820 \text{ psi} \).

(a) Find the principal stresses. Show them on a sketch of a properly oriented element.

(b) Find the maximum shear stresses and associated normal stresses. Show them on a sketch of a properly oriented element.

7.3-12 through 7.3-15 An element in plane stress (see figure) is subjected to stresses \( \sigma_x, \sigma_y, \) and \( \tau_{xy} \).

(a) Determine the principal stresses and show them on a sketch of a properly oriented element.

(b) Determine the maximum shear stresses and associated normal stresses and show them on a sketch of a properly oriented element.

Problems 7.3-12 through 7.3-15

7.3-12 \( \sigma_x = 2150 \text{ kPa}, \sigma_y = 375 \text{ kPa}, \tau_{xy} = -460 \text{ kPa} \)

7.3-13 \( \sigma_x = 14,500 \text{ psi}, \sigma_y = 1070 \text{ psi}, \tau_{xy} = 1900 \text{ psi} \)

7.3-14 \( \sigma_x = 16.5 \text{ MPa}, \sigma_y = -91 \text{ MPa}, \tau_{xy} = -39 \text{ MPa} \)

7.3-15 \( \sigma_x = -3300 \text{ psi}, \sigma_y = -11,000 \text{ psi}, \tau_{xy} = 4500 \text{ psi} \)

7.3-16 \( \sigma_x = -108 \text{ MPa}, \sigma_y = 58 \text{ MPa}, \tau_{xy} = -58 \text{ MPa} \)
7.3-17 At a point on the web of a girder on a gantry crane, the stresses acting on the x face of a stress element are \( \sigma_x = 6250 \) psi and \( \tau_{xy} = 1425 \) psi (see figure).

What is the allowable range of values for the stress \( \sigma_x \) if the maximum shear stress is limited to \( \tau_0 = 2150 \) psi?

PROB. 7.3-17 (ZCW/Shutterstock)

7.3-18 The stresses acting on a stress element on the arm of a power excavator (see figure) are \( \sigma_x = 52 \) MPa and \( \tau_{xy} = 33 \) MPa (see figure).

What is the allowable range of values for the stress \( \sigma_x \) if the maximum shear stress is limited to \( \tau_0 = 37 \) MPa?

PROB. 7.3-18 (Can Stock Photo Inc./busja)

7.3-19 The stresses at a point on the down tube of a bicycle frame are \( \sigma_y = 4800 \) psi and \( \tau_{yx} = -1950 \) psi (see figure). It is known that one of the principal stresses equals 6375 psi in tension.

(a) Determine the stress \( \sigma_y \).

(b) Determine the other principal stress and the orientation of the principal planes, then show the principal stresses on a sketch of a properly oriented element.

PROB. 7.3-19 (Can Stock Photo Inc./Aviafan)

7.3-20 An element in plane stress on the surface of an automobile drive shaft (see figure) is subjected to stresses of \( \sigma_x = -45 \) MPa and \( \tau_{xy} = 39 \) MPa (see figure). It is known that one of the principal stresses equals 41 MPa in tension.

(a) Determine the stress \( \sigma_x \).

(b) Determine the other principal stress and the orientation of the principal planes, then show the principal stresses on a sketch of a properly oriented element.

PROB. 7.3-20 (Courtesy of www.rietzusa.com)

Mohr's Circle

The problems for Section 7.4 are to be solved using Mohr's circle. Consider only the in-plane stresses (the stresses in the xy plane).

7.4-1 An element in uniaxial stress is subjected to tensile stresses \( \sigma_x = 14,250 \) psi, as shown in the figure. Using Mohr's circle, determine the following.

(a) The stresses acting on an element oriented at \( \theta = 29^\circ \) from the x axis.

(b) The maximum shear stresses and associated normal stresses.

Show all results on sketches of properly oriented elements.

PROB. 7.4-1

7.4-2 An element in uniaxial stress is subjected to tensile stresses \( \sigma_x = 57 \) MPa, as shown in the figure. Using Mohr's circle, determine the following.

(a) The stresses acting on an element oriented at \( \angle = -33^\circ \) from the x axis (minus means clockwise)

(b) The maximum shear stresses and associated normal stresses.
Show all results on sketches of properly oriented elements.

PROB. 7.4-2

7.4-3 An element on the gusset plate in Prob. 7.2-17 in uniaxial stress is subjected to compressive stresses of magnitude 6750 psi, as shown in the figure. Using Mohr's circle, determine the following.

(a) The stresses acting on an element oriented at a slope of 1 on 2 (see figure).

(b) The maximum shear stresses and associated normal stresses.

Show all results on sketches of properly oriented elements.

PROB. 7.4-3

7.4-4 An element on the top surface of the fuel tanker in Prob. 7.2-1 is in biaxial stress and is subjected to stresses $\sigma_x = -48$ MPa and $\sigma_y = 19$ MPa, as shown in the figure. Using Mohr's circle, determine the following.

(a) The stresses acting on an element oriented at a counterclockwise angle $\theta = 25^\circ$ from the x axis.

(b) The maximum shear stresses and associated normal stresses.

Show all results on sketches of properly oriented elements.

PROB. 7.4-4

7.4-5 An element on the top surface of the fuel tanker in Prob. 7.2-1 is in biaxial stress and is subjected to stresses $\sigma_x = 6250$ psi and $\sigma_y = -1750$ psi, as shown in the figure. Using Mohr's circle, determine the following.

(a) The stresses acting on an element oriented at a counterclockwise angle $\theta = 55^\circ$ from the x axis.

(b) The maximum shear stresses and associated normal stresses.

Show all results on sketches of properly oriented elements.

PROB. 7.4-5

7.4-6 An element in biaxial stress is subjected to stresses $\sigma_x = -29$ MPa and $\sigma_y = 57$ MPa, as shown in the figure. Using Mohr's circle, determine the following.

(a) The stresses acting on an element oriented at a slope of 1 on 2.5 (see figure).

(b) The maximum shear stresses and associated normal stresses.

Show all results on sketches of properly oriented elements.

PROB. 7.4-6
7.4-7 An element on the surface of a drive shaft is in pure shear and is subjected to stresses $\tau_{xy} = 2700$ psi, as shown in the figure. Using Mohr's circle, determine the following:

(a) The stresses acting on an element oriented at a counterclockwise angle $\theta = 52^\circ$ from the $x$ axis.

(b) The principal stresses.

Show all results on sketches of properly oriented elements.

![Diagram of pure shear stresses](image)

PROB 7.4-7

7.4-8 The rotor shaft of a helicopter (see figure part a) drives the rotor blades that provide the lifting force and is subjected to a combination of torsion and axial loading (see figure part b).

It is known that normal stress $\sigma_x = 68$ MPa and shear stress $\tau_{xy} = -100$ MPa. Using Mohr's circle, determine the following:

(a) The stresses acting on an element oriented at a counterclockwise angle $\theta = 22.5^\circ$ from the $x$ axis

(b) Find the maximum tensile stress, maximum compressive stress, and maximum shear stress in the shaft.

Show all results on sketches of properly oriented elements.

![Diagram of helicopter rotor shaft](image)

PROB 7.4-8

7.4-9 An element in plane stress is subjected to stresses $\sigma_x$, $\sigma_y$, and $\tau_{xy}$ (see figure).

Using Mohr's circle, determine the stresses acting on an element oriented at an angle $\theta$ from the $x$ axis. Show these stresses on a sketch of an element oriented at the angle $\theta$. (Note: The angle $\theta$ is positive when counterclockwise and negative when clockwise.)

![Diagram of plane stress](image)

PROBS 7.4-10 through 7.4-15

7.4-10 $\sigma_x = 27$ MPa, $\sigma_y = 14$ MPa, $\tau_{xy} = 6$ MPa, $\theta = 40^\circ$

7.4-11 $\sigma_x = 3500$ psi, $\sigma_y = 12200$ psi, $\tau_{xy} = -3300$ psi, $\theta = -51^\circ$

7.4-12 $\sigma_x = -47$ MPa, $\sigma_y = -186$ MPa, $\tau_{xy} = -29$ MPa, $\theta = -33^\circ$

7.4-13 $\sigma_x = -1720$ psi, $\sigma_y = -680$ psi, $\tau_{xy} = 320$ psi, $\theta = 14^\circ$

7.4-14 $\sigma_x = 33$ MPa, $\sigma_y = -9$ MPa, $\tau_{xy} = 29$ MPa, $\theta = 35^\circ$

7.4-15 $\sigma_x = -5700$ psi, $\sigma_y = 950$ psi, $\tau_{xy} = -2100$ psi, $\theta = 65^\circ$
An element in plane stress is subjected to stresses $\sigma_x$, $\sigma_y$, and $\tau_{xy}$ (see figure). Using Mohr's circle, determine (a) the principal stresses, and (b) the maximum shear stresses and associated normal stresses. Show all results on sketches of properly oriented elements.

![Diagram of plane stress](image)

ProBS 7.4-18 through 7.4-23

7.4-16 $\sigma_x = 2900$ kPa, $\sigma_y = 9100$ kPa, $\tau_{xy} = -3750$ kPa
7.4-17 $\sigma_x = 800$ psi, $\sigma_y = -2200$ psi, $\tau_{xy} = 2900$ psi
7.4-18 $\sigma_x = -3.3$ MPa, $\sigma_y = 8.9$ MPa, $\tau_{xy} = -14.1$ MPa
7.4-19 $\sigma_x = -11500$ psi, $\sigma_y = -18250$ psi, $\tau_{xy} = -7200$ psi
7.2-19 $\sigma_x = -29.5$ MPa, $\sigma_y = 29.5$ MPa, $\tau_{xy} = 27$ MPa
7.6-19 $\sigma_x = 2050$ psi, $\sigma_y = 6100$ psi, $\tau_{xy} = 2750$ psi
7.6-21 $\sigma_x = 0$ MPa, $\sigma_y = -23.4$ MPa, $\tau_{xy} = -9.6$ MPa
7.6-22 $\sigma_x = 7300$ psi, $\sigma_y = 0$ psi, $\tau_{xy} = 1300$ psi

ProBS 7.5-1 and 7.5-2

7.5-2 Solve the preceding problem if the thickness of the steel plate is $t = 12$ mm, the gage readings are $\varepsilon_x = 530 \times 10^{-6}$ (elongation) and $\varepsilon_y = -210 \times 10^{-6}$ (shortening), the modulus is $E = 200$ GPa, and Poisson's ratio is $\nu = 0.30$.

7.5-3 Assume that the normal strains $\varepsilon_x$ and $\varepsilon_y$ for an element in plane stress (see figure) are measured with strain gages.

(a) Obtain a formula for the normal strain $\varepsilon_z$ in the $z$-direction in terms of $\varepsilon_x$, $\varepsilon_y$, and Poisson's ratio $\nu$.

(b) Obtain a formula for the dilatation $\varepsilon$ in terms of $\varepsilon_x$, $\varepsilon_y$, and Poisson's ratio $\nu$.

Hook's Law for Plane Stress

When solving the problems for Section 7.5, assume that the material is linearly elastic with modulus of elasticity $E$ and Poisson's ratio $\nu$.

7.5-1 A rectangular steel plate with thickness $t = 5/8$ in. is subjected to uniform normal stresses $\sigma_x$ and $\sigma_y$, as shown in the figure. Strain gages A and B, oriented in the $x$ and $y$ directions, respectively, are attached to the plate. The gage readings give normal strains $\varepsilon_x = 0.00065$ (elongation) and $\varepsilon_y = 0.00040$ (elongation).

Knowing that $E = 30 \times 10^6$ psi and $\nu = 0.3$, determine the stresses $\sigma_x$ and $\sigma_y$ and the change $\Delta \sigma$ in the thickness of the plate.
7.5-4 A cast-iron plate in biaxial stress is subjected to tensile stresses \( \sigma_x = 31 \text{ MPa} \) and \( \sigma_y = 17 \text{ MPa} \) (see figure). The corresponding strains in the plate are \( \varepsilon_x = 240 \times 10^{-6} \) and \( \varepsilon_y = 85 \times 10^{-6} \).

Determine Poisson's ratio \( \nu \) and the modulus of elasticity \( E \) for the material.

PROBS 7.5-4 through 7.5-7

7.5-5 Solve the preceding problem for a steel plate with \( \sigma_x = 11,600 \text{ psi (tension)}, \sigma_y = -5700 \text{ psi (compression)} \), \( \varepsilon_x = 450 \times 10^{-6} \) (elongation), and \( \varepsilon_y = -310 \times 10^{-6} \) (shortening).

7.5-6 A rectangular plate in biaxial stress (see figure) is subjected to normal stresses \( \sigma_x = 67 \text{ MPa (tension)} \) and \( \sigma_y = -23 \text{ MPa (compression)} \). The plate has dimensions \( 400 \times 550 \times 20 \text{ mm} \) and is made of steel with \( E = 200 \text{ GPa} \) and \( \nu = 0.30 \).

(a) Determine the maximum in-plane shear \( \gamma_{\text{max}} \) in the plate.

(b) Determine the change \( \Delta t \) in the thickness of the plate.

(c) Determine the change \( \Delta V \) in the volume of the plate.

7.5-7 Solve the preceding problem for an aluminum plate with \( \sigma_x = 12,000 \text{ psi (tension)}, \sigma_y = -3000 \text{ psi (compression)} \), dimensions \( 20 \times 30 \times 0.5 \text{ in.} \), \( E = 10.5 \times 10^6 \text{ psi} \), and \( \nu = 0.33 \).

7.5-8 A brass cube of 48 mm on each edge is compressed in two perpendicular directions by forces \( P = 160 \text{ kN} \) (see figure).

(a) Calculate the change \( \Delta V \) in the volume of the cube and the strain energy \( U \) stored in the cube, assuming \( E = 100 \text{ GPa} \) and \( \nu = 0.34 \).

(b) Repeat part (a) if the cube is made of an aluminum alloy with \( E = 73 \text{ GPa} \) and \( \nu = 0.33 \).

PROB 7.5-8

7.5-9 A 4.0-inch cube of concrete (\( E = 4.5 \times 10^6 \text{ psi}, \nu = 0.2 \)) is compressed in biaxial stress by means of a framework that is loaded as shown in the figure.

Assuming that each load \( F \) equals 25 k, determine the change \( \Delta V \) in the volume of the cube and the strain energy \( U \) stored in the cube.

PROB 7.5-9

7.5-10 A square plate of width \( b \) and thickness \( t \) is loaded by normal forces \( P_x \) and \( P_y \) and by shear forces \( V \), as shown in the figure. These forces produce uniformly distributed stresses acting on the side faces of the plate.

(a) Calculate the change \( \Delta V \) in the volume of the plate and the strain energy \( U \) stored in the plate if the dimensions are \( b = 600 \text{ mm and } t = 40 \text{ mm} \); the plate is made of magnesium with \( E = 41 \text{ GPa} \) and \( \nu = 0.35 \); and the forces are \( P_x = 420 \text{ kN}, P_y = 210 \text{ kN} \), and \( V = 96 \text{ kN} \).

(b) Find the maximum permissible thickness of the plate when the strain energy \( U \) must be at least 62 J. (Assume that all other numerical values in part (a) are unchanged.)

(c) Find the minimum with \( b \) of the square plate of thickness \( t = 40 \text{ mm} \) when the change in volume of the plate cannot exceed 0.018% of the original volume.
\textbf{Triaxial Stress}

When solving the problems for Section 7.6, assume that the material is linearly elastic with modulus of elasticity \( E \) and Poisson's ratio \( \nu \).

\textbf{7.6.1} An element of aluminum in the form of a rectangular parallelepiped (see figure) of dimensions \( a = 5.5 \text{ in.}, \ b = 4.5 \text{ in.}, \) and \( c = 3.5 \text{ in.} \) is subjected to triaxial stresses \( \sigma_x = 12,500 \text{ psi}, \ \sigma_y = -5000 \text{ psi}, \) and \( \sigma_z = -1400 \text{ psi} \) acting on the \( x, \ y, \) and \( z \) faces, respectively.

Determine the following quantities: (a) the maximum shear stress \( \tau_{\text{max}} \) in the material; (b) the changes \( \Delta a, \Delta b, \) and \( \Delta c \) in the dimensions of the element; (c) the change \( \Delta V \) in the volume; (d) the strain energy \( U \) stored in the element; (e) the maximum value of \( \sigma_z \) when the change in volume must be limited to 0.021%; and (f) the required value of \( \sigma_x \) when the strain energy must be 900 in.-lb. (Assume \( E = 10,400 \text{ ksi} \) and \( \nu = 0.33 \).)

\textbf{7.5.12} A circle of diameter \( d = 200 \text{ mm} \) is etched on a brass plate (see figure). The plate has dimensions of \( 400 \times 400 \times 20 \text{ mm} \). Forces are applied to the plate, producing uniformly distributed normal stresses \( \sigma_x = 59 \text{ MPa} \) and \( \sigma_y = -17 \text{ MPa} \). Calculate the following quantities: (a) the change in length \( \Delta ac \) of diameter \( ac \); (b) the change in length \( \Delta bd \) of diameter \( bd \); (c) the change \( \Delta t \) in the thickness of the plate; (d) the change \( \Delta V \) in the volume of the plate, and (e) the strain energy \( U \) stored in the plate; (f) the maximum permissible thickness of the plate when strain energy \( U \) must be at least 78.4 J; (g) the maximum permissible value of normal stress \( \sigma_z \) when the change in volume of the plate cannot exceed 0.015\% of the original volume. (Assume \( E = 100 \text{ GPa} \) and \( \nu = 0.34 \).)

\textbf{7.6.2} Solve the preceding problem if the element is steel \( (E = 200 \text{ GPa}, \ \nu = 0.30) \) with dimensions \( a = 300 \text{ mm}, \ b = 150 \text{ mm}, \) and \( c = 150 \text{ mm} \) and the stresses \( \sigma_x = -62 \text{ MPa}, \ \sigma_y = -45 \text{ MPa}, \) and \( \sigma_z = -45 \text{ MPa} \).

For part (e), find the maximum value of \( \sigma_x \) if the change in volume must be limited to \(-0.028\%\). For part (f), find the required value of \( \sigma_x \) if the strain energy must be 60 J.
Chapter 7  Analysis of Stress and Strain

7.6.3 A cube of cast iron with sides of length \( a = 4.0 \text{ in.} \) (see figure) is tested in a laboratory under triaxial stress. Gages mounted on the testing machine show that the compressive strains in the material are \( \varepsilon_x = -225 \times 10^{-6} \) and \( \varepsilon_y = \varepsilon_z = -37.5 \times 10^{-6} \).

Determine the following quantities: (a) the normal stresses \( \sigma_x, \sigma_y, \) and \( \sigma_z \) acting on the \( x, y, \) and \( z \) faces of the cube; (b) the maximum shear stress \( \tau_{\text{max}} \) in the material; (c) the change \( \Delta V \) in the volume of the cube; (d) the strain energy \( U \) stored in the cube; (e) the maximum value of \( \sigma_x \) when the change in volume must be limited to 0.028%; and (f) the required value of \( \varepsilon_x \) when the strain energy must be 38 in.-lb. (Assume \( E = 14,000 \text{ ksi} \) and \( \nu = 0.25 \)).

7.6.4 Solve the preceding problem if the cube is granite (\( E = 80 \text{ GPa}, \nu = 0.25 \)) with dimensions \( a = 89 \text{ mm} \) and compressive strains \( \varepsilon_x = 690 \times 10^{-6} \) and \( \varepsilon_y = \varepsilon_z = 255 \times 10^{-6} \). For part (e), find the maximum value of \( \sigma_x \) when the change in volume must be limited to 0.11%. For part (f), find the required value of \( \varepsilon_x \) when the strain energy must be 33 J.

7.6.5 An element of aluminum is subjected to triaxial stress (see figure).

(a) Find the bulk modulus \( K \) for the aluminum if the following stress and strain data is known: normal stresses are \( \sigma_x = 5200 \text{ psi} \) (tension), \( \sigma_y = -4750 \text{ psi} \) (compression), and \( \sigma_z = -3090 \text{ psi} \) (compression) and normal strains in the \( x \) and \( y \) directions are \( \varepsilon_x = 713.8 \times 10^{-6} \) (elongation) and \( \varepsilon_y = -502.3 \times 10^{-6} \) (shortening).

(b) If the element is replaced by one of magnesium, find the modulus of elasticity \( E \) and Poisson’s ratio \( \nu \) if the following data is given: bulk modulus \( K = 6.8 \times 10^6 \text{ psi} \); normal stresses are \( \sigma_x = 4550 \text{ psi} \) (tension), \( \sigma_y = -1700 \text{ psi} \) (compression), and \( \sigma_z = -1090 \text{ psi} \) (compression); and normal strain in the \( x \) direction is \( \varepsilon_x = 900 \times 10^{-6} \) (elongation).

7.6.6 Solve the preceding problem if the material is nylon.

(a) Find the bulk modulus \( K \) for the nylon if the following stress and strain data is known: normal stresses are \( \sigma_x = -3.9 \text{ MPa} \), \( \sigma_y = -3.2 \text{ MPa} \), and \( \sigma_z = -1.8 \text{ MPa} \); and normal strains in the \( x \) and \( y \) directions are \( \varepsilon_x = -640 \times 10^{-6} \) (shortening) and \( \varepsilon_y = -310 \times 10^{-6} \) (shortening).

(b) If the element is replaced by one of polyethylene, find the modulus of elasticity \( E \) and Poisson’s ratio \( \nu \) if the following data is given: bulk modulus \( K = 2162 \text{ MPa} \); normal stresses are \( \sigma_x = -3.6 \text{ MPa} \) (compression), \( \sigma_y = -2.1 \text{ MPa} \) (compression), and \( \sigma_z = -2.1 \text{ MPa} \) (compression); and normal strain in the \( x \) direction is \( \varepsilon_x = -1480 \times 10^{-6} \) (shortening).

7.6.7 A rubber cylinder \( R \) of length \( L \) and cross-sectional area \( A \) is compressed inside a steel cylinder \( S \) by a force \( F \) that applies a uniformly distributed pressure to the rubber (see figure).

(a) Derive a formula for the lateral pressure \( p \) between the rubber and the steel. (Disregard friction between the rubber and the steel, and assume that the steel cylinder is rigid when compared to the rubber.)

(b) Derive a formula for the shortening \( \delta \) of the rubber cylinder.
5-8 A block \( R \) of rubber is confined between plane parallel walls of a steel block \( S \) (see figure). A uniformly distributed pressure \( p_0 \) is applied to the top of the rubber block by a force \( F \).

(a) Derive a formula for the lateral pressure \( p \) between the rubber and the steel. (Disregard friction between the rubber and the steel, and assume that the steel block is rigid when compared to the rubber.)

(b) Derive a formula for the dilatation \( e \) of the rubber.

(c) Derive a formula for the strain-energy density \( u \) of the rubber.

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**Plane Strain**

When solving the problems for Section 7.7, consider only the in-plane strains (the strains in the \( xy \) plane) unless stated otherwise. Use the transformation equations of plane strain except when Mohr’s circle is specified (Prob. 7.7-23 through 7.7-28).

7.7.1 A thin rectangular plate in biaxial stress is subjected to stresses \( \sigma_x \) and \( \sigma_y \), as shown in part a of the figure. The width and height of the plate are \( b = 7.5 \) in. and \( h = 2.5 \) in., respectively. Measurements show that the normal strains in the \( x \) and \( y \) directions are \( \varepsilon_x = 285 \times 10^{-6} \) and \( \varepsilon_y = -190 \times 10^{-6} \), respectively.

With reference to part b of the figure, which shows a two-dimensional view of the plate, determine the following quantities.

(a) The increase \( \Delta d \) in the length of diagonal \( Od \).

(b) The change \( \Delta \phi \) in the angle \( \phi \) between diagonal \( Od \) and the \( x \) axis.

(c) The change \( \Delta \psi \) in the angle \( \psi \) between diagonal \( Od \) and the \( y \) axis.

7.7.9 A solid spherical ball of magnesium alloy \((E = 6.5 \times 10^6 \) psi, \( \nu = 0.35 \)) is lowered into the ocean to a depth of 8000 ft. The diameter of the ball is 9.0 in.

(a) Determine the decrease \( \Delta d \) in diameter, the decrease \( \Delta V \) in volume, and the strain energy \( U \) of the ball.

(b) At what depth will the volume change be equal to 0.0324% of the original volume?

7.7.10 A solid steel sphere \((E = 210 \) GPa, \( \nu = 0.3 \)) is subjected to hydrostatic pressure \( p \) such that its volume is reduced by 0.4%.

(a) Calculate the pressure \( p \).

(b) Calculate the volume modulus of elasticity \( K \) for the steel.

(c) Calculate the strain energy \( U \) stored in the sphere if its diameter is \( d = 150 \) mm.

7.7.11 A solid bronze sphere (volume modulus of elasticity \( K = 14.5 \times 10^6 \) psi) is suddenly heated around its outer surface. The tendency of the heated part of the sphere to expand produces uniform tension in all directions at the center of the sphere.

If the stress at the center is 12,000 psi, what is the strain? Also, calculate the unit volume change \( e \) and the strain-energy density \( u \) at the center.

PROBS 7.71 and 7.72

7.72 Solve the preceding problem if \( b = 180 \) mm, and \( h = 70 \) mm, respectively. Measurements show that the normal strains in the \( x \) and \( y \) directions are \( \varepsilon_x = 390 \times 10^{-6} \) and \( \varepsilon_y = -240 \times 10^{-6} \), respectively.