4.3-1 Calculate the shear force $V$ and bending moment $M$ at a cross section just to the left of the 1600-lb load acting on the simple beam $AB$ shown in the figure.

PROB. 4.3-1

4.3-2 Determine the shear force $V$ and bending moment $M$ at the midpoint $C$ of the simple beam $AB$ shown in the figure.

PROB. 4.3-2

4.3-3 Determine the shear force $V$ and bending moment $M$ at the midpoint of the beam with overhangs (see figure). Note that one load acts downward and the other upward, and clockwise moments $P_E$ are applied at each support.

PROB. 4.3-3

4.3-4 Calculate the shear force $V$ and bending moment $M$ at a cross section located 0.5 m from the fixed support of the cantilever beam $AB$ shown in the figure.

PROB. 4.3-4

4.3-5 Consider the beam with an overhang shown in the figure.

(a) Determine the shear force $V$ and bending moment $M$ at a cross section located 18 ft from the left-hand end $A$.

(b) Find the required magnitude of load intensity $q$ acting on the right half of member BC that will result in a zero shear force on the cross section 18 ft from $A$.

PROB. 4.3-5

4.3-6 The beam ABC shown in the figure is simply supported at $A$ and $B$ and has an overhang from $B$ to $C$. The loads consist of a horizontal force $P_1 = 4.0$ kN acting at the end of a vertical arm and a vertical force $P_2 = 8.0$ kN acting at the end of the overhang.

(a) Determine the shear force $V$ and bending moment $M$ at a cross section located 3.0 m from the left-hand support. (Note: Disregard the widths of the beam and vertical arm and use centerline dimensions when making calculations.)

(b) Find the value of load $P_2$ that results in $V = 0$ at a cross section located 2.0 m from the left-hand support.

(c) If $P_2 = 8$ kN, find the value of load $P_1$ that results in $M = 0$ at a cross section located 2.0 m from the left-hand support.
4.3.7 The beam \( ABCD \) shown in the figure has overhangs at each end and carries a uniform load of intensity \( q \).

For what ratio \( b/L \) will the bending moment at the midpoint of the beam be zero?

4.3.8 At full draw, an archer applies a pull of 130 N to the bowstring of the bow shown in the figure. Determine the bending moment at the midpoint of the bow.

4.3.9 A curved bar \( ABC \) is subjected to loads in the form of two equal and opposite forces \( P \), as shown in the figure. The axis of the bar forms a semicircle of radius \( r \).

Determine the axial force \( N \), shear force \( V \), and bending moment \( M \) acting at a cross section defined by the angle \( \theta \).

4.3.10 Under cruising conditions the distributed load acting on the wing of a small airplane has the idealized variation shown in the figure.

Calculate the shear force \( V \) and bending moment \( M \) at the inboard end of the wing.

Wings of a small airplane have distributed uplift loads. (Thomasz Gulla/Shutterstock)
4.3-11 A beam *ABCD* with a vertical arm *CE* is supported as a simple beam at *A* and *D* (see figure part a). A cable passes over a small pulley that is attached to the arm at *E*. One end of the cable is attached to the beam at point *B*.

(a) What is the force *P* in the cable if the bending moment in the beam just to the left of point *C* is equal numerically to 640 lb-ft? (Note: Disregard the widths of the beam and vertical arm and use centerline dimensions when making calculations.)

(b) Repeat part (a) if a roller support is added at *C* and a shear release is inserted just left of *C* (see figure part b).

4.3-13 Beam *ABCD* represents a reinforced-concrete foundation beam that supports a uniform load of intensity *q_1* = 3500 lb/ft (see figure). Assume that the soil pressure on the underside of the beam is uniformly distributed with intensity *q_2*.

(a) Find the shear force *V_B* and bending moment *M_B* at point *B*.

(b) Find the shear force *V_m* and bending moment *M_m* at the midpoint of the beam.

![Beam Diagram](image)

**PROB 4.3-13**

4.3-14 The simply supported beam *ABCD* is loaded by a weight *W* = 27 kN through the arrangement shown in the figure part a. The cable passes over a small frictionless pulley at *B* and is attached at *E* to the end of the vertical arm.

(a) Calculate the axial force *N*, shear force *V*, and bending moment *M* at section *C*, which is just to the left of the vertical arm. (Note: Disregard the widths of the beam and vertical arm and use centerline dimensions when making calculations.)

(b) Repeat part (a) if a roller support is added at *C* and a moment release is inserted just left of *C* (see figure part b).

![Beam Diagram](image)

**PROB 4.3-14**
4.3-15 The centrifuge shown in the figure rotates in a horizontal plane (the xy plane) on a smooth surface about the z axis (which is vertical) with an angular acceleration α. Each of the two arms has weight w per unit length and supports a weight $W = 2.0wL$ at its end.

Derive formulas for the maximum shear force and maximum bending moment in the arms, assuming $b = L/9$ and $c = L/10$.

**PROB. 4.3-15**

**Shear-Force and Bending-Moment Diagrams**

When solving the problems for Section 4.5, draw the shear-force and bending-moment diagrams approximately to scale and label all critical ordinates, including the maximum and minimum values.

Probs. 4.5-1 through 4.5-10 are symbolic problems and Probs. 4.5-11 through 4.5-24 are numerical problems. The remaining problems (4.5-25 through 4.5-40) involve specialized topics, such as optimization, beams with hinges, and moving loads.

4.5-1 Draw the shear-force and bending-moment diagrams for a simple beam $AB$ supporting two equal concentrated loads $P$ (see figure).

**PROB. 4.5-1**

4.5-2 A simple beam $AB$ is subjected to a counterclockwise couple of moment $M_0$ acting at distance $a$ from the left-hand support (see figure).

**PROB. 4.5-2**

Draw the shear-force and bending-moment diagrams for this beam.

4.5-3 Draw the shear-force and bending-moment diagrams for a cantilever beam $AB$ carrying a uniform load of intensity $q$ over one-half of its length (see figure).

**PROB. 4.5-3**

4.5-4 The cantilever beam $AB$ shown in the figure is subjected to a concentrated load $P$ at the midpoint and a counterclockwise couple of moment $M_1 = PL/4$ at the free end.

Draw the shear-force and bending-moment diagrams for this beam.

**PROB. 4.5-4**

4.5-5 The simple beam $AB$ shown in the figure is subjected to a concentrated load $P$ and a clockwise couple $M_1 = PL/3$ acting at the third points.

Draw the shear-force and bending-moment diagrams for this beam.

**PROB. 4.5-5**
4.5-6 A simple beam \( AB \) subjected to couples \( M_1 \) and \( 3M_1 \) acting at the third points is shown in the figure.

Draw the shear-force and bending-moment diagrams for this beam.

4.5-9 Beam \( ABCD \) is simply supported at \( B \) and \( C \) and has overhangs at each end (see figure). The span length is \( L \) and each overhang has length \( L/3 \). A uniform load of intensity \( q \) acts along the entire length of the beam.

Draw the shear-force and bending-moment diagrams for this beam.

4.5-7 A simply supported beam \( ABC \) is loaded by a vertical load \( P \) acting at the end of a bracket \( BDE \) (see figure).

(a) Draw the shear-force and bending-moment diagrams for beam \( ABC \).

(b) Now assume that load \( P \) at \( E \) is directed to the right. The vertical dimension \( BD \) is \( L/5 \). Draw axial-force, shear-force, and bending-moment diagrams for \( ABC \).

4.5-10 Draw the shear-force and bending-moment diagrams for a cantilever beam \( AB \) acted upon by two different load cases.

(a) A distributed load with linear variation and maximum intensity \( q_0 \) (see figure part a).

(b) A distributed load with parabolic variation and maximum intensity \( q_0 \) (see figure part b).

4.5-8 A beam \( ABC \) is simply supported at \( A \) and \( B \) and has an overhang \( BC \) (see figure). The beam is loaded by two forces \( P \) and a clockwise couple of moment \( Pa \) at \( D \) that act through the arrangement shown.

(a) Draw the shear-force and bending-moment diagrams for beam \( ABC \).

(b) If moment \( Pa \) at \( D \) is replaced by moment \( M \), find an expression for \( M \) in terms of variables \( P \) and \( a \) so that the reaction at \( B \) goes to zero. Plot the associated shear-force and bending-moment diagrams for beam \( ABC \).

4.5-11 The simple beam \( AB \) supports a triangular load of maximum intensity \( q_0 = 10 \text{ lb/in} \) acting over one-half of the span and a concentrated load \( P = 80 \text{ lb} \) acting at midspan (see figure). Draw the shear-force and bending-moment diagrams for this beam.
4.5-11 The beam $AB$ shown in the figure supports a uniform load of intensity $3000 \text{ N/m}$ acting over half the length of the beam. The beam rests on a foundation that produces a uniformly distributed load over the entire length. Draw the shear-force and bending-moment diagrams for this beam.

4.5-12 A cantilever beam $AB$ supports a couple and a concentrated load, as shown in the figure. Draw the shear-force and bending-moment diagrams for this beam.

4.5-13 The cantilever beam $AB$ shown in the figure is subjected to a triangular load acting throughout one-half of its length and a concentrated load acting at the free end. Draw the shear-force and bending-moment diagrams for this beam.

4.5-14 The uniformly loaded beam $ABC$ has simple supports at $A$ and $B$ and an overhang $BC$ (see figure). Draw the shear-force and bending-moment diagrams for this beam.

4.5-16 A beam $ABC$ with an overhang at one end supports a uniform load of intensity $12 \text{ kN/m}$ and a concentrated moment of magnitude $3 \text{ kN} \cdot \text{m}$ at $C$ (see figure). Draw the shear-force and bending-moment diagrams for this beam.

4.5-17 Consider two beams, which are loaded the same but have different support conditions. Which beam has the larger maximum moment?

First, find support reactions, then plot axial force ($N$), shear ($V$), and moment ($M$) diagrams for all three beams. Label all critical $N$, $V$, and $M$ values and also the distance to points where $N$, $V$, and/or $M$ is zero.
Chapter 4  Shear Forces and Bending Moments

4.5-18 The three beams below are loaded the same and have the same support conditions. However, one has a moment release just to the left of C, the second has a shear release just to the right of C and the third has an axial release just to the left of C. Which beam has the largest maximum moment?

First, find support reactions, then plot axial force (N), shear (V), and moment (M) diagrams for all three beams. Label all critical N, V, and M values and also the distance to points where N, V, and/or M is zero.

![Diagram of three beams with loads and reactions](image)

4.5-19 The beam ABC shown in the figure is simply supported at A and B and has an overhang from B to C. The loads consist of a horizontal force \( P_1 = 400 \text{ lb} \) acting at the end of the vertical arm and a vertical force \( P_2 = 900 \text{ lb} \) acting at the end of the overhang.

![Diagram of beam ABC](image)

4.5-20 A simple beam \( AB \) is loaded by two segments - uniform load and two horizontal forces acting at the end of a vertical arm (see figure).

Draw the shear-force and bending-moment diagram for this beam.

![Diagram of beam AB](image)

4.5-21 The two beams below are loaded the same and have the same support conditions. However, the location of internal axial, shear, and moment releases is different for each beam (see figures). Which beam has the larger maximum moment?

First, find support reactions, then plot axial force (N), shear (V), and moment (M) diagrams for both beams. Label all critical N, V, and M values and also the distance to points where N, V, and/or M is zero.

![Diagram of two beams with loads and reactions](image)