The amount and distribution of moisture in soil have a very great effect on the physical properties of the soil. The ability of a soil to support foundation loads may vary over a range of several hundred percent, depending on the percent of moisture and the type of soil. Seepage flow through and below an earth dam must be considered in evaluating the dam’s ability to impound water. The stability of artificial and natural slopes is greatly affected by the presence of water pressure. Because of these and many other reasons, it is, therefore, necessary to have an accurate and complete knowledge of the water conditions in the soil. The following sections are devoted to an understanding of this problem.

**STATIC PRESSURES IN WATER**

The pressure in a static body of water (see Fig. 46-1a) has a triangular distribution with a magnitude $\gamma_{w}y$. A pressure conforming to this definition is called hydrostatic pressure (Fig. 46-1a). A hydrostatic pressure distribution also exists in the pore water surrounding the soil particles in Fig. 46-1b. If, however, the particles are suspended in the water or are falling with a constant velocity (Fig. 46-1c), the magnitude of the pressure at any point below the surface may be found by the equation $u = \gamma y$ (see Fig. 46-1), where $\gamma$ is the unit weight of the soil-water combination. The computation of static water pressure is illustrated in Fig. 46-2.
Figure 46-1  Static water pressure.

(a) Water

(b) Water surrounding soil particles

(c) Suspended particles in water

Figure 46-2

Example Problem  Find the water pressure ($u$) at the bottom of the container in Fig. 46-1a ($y = 8$ m). Where $u =$ water pressure and $\gamma_w =$ Unit weight of water.

Solution  

\[ u = \gamma_w y = 9810 \text{ N/m}^2 \times 8 \text{ m} \]

\[ = 78.5 \text{ kPa} \]
CAPILLARY MOISTURE

The water zone in a soil mass that has a water table may be divided into a saturated zone below the water table and a capillary zone above the water table. Some voids are filled with air in the saturated zone, but these air voids have little effect on the reaction of the soil to outside stresses. Above the water table, the number of air voids increases as the distance from the water table increases. The capillary zone may be divided into three different zones with somewhat arbitrary boundaries (see Fig. 47-1). The zone closest to the water table is called the zone of capillary saturation. The water content in this zone may be slightly less than 100%, but forces exerted on the soil structure by capillarity are very small and the soil reacts much as if it were in the saturated condition. Above the zone of capillary saturation is the zone of partial capillary saturation. In this zone water is connected through the smaller pores, but more of the larger pores are filled with air. The third zone is the zone of contact water. The water in this zone surrounds the points of contact between soil particles and also surrounds soil particles, but is disconnected through the pores. The water in the capillary zones

Figure 47-1  Capillary water system.
Figure 48-1 Height of rise in a capillary tube.

\[ W = F_T \cos \alpha \text{ (equilibrium of water column)} \]
\[ \gamma \pi r^2 h = T_s (2\pi r) \cos \alpha \]

where

\[ T_s = \text{surface tension for water} = 0.0735 \text{ N/m (water)} \]

The height of capillary rise may be expressed as

\[ h = \frac{2T_s}{\gamma \pi} \cos \alpha \quad (48-1a) \]

and the pressure difference across the air water interface may be expressed as

\[ u_H = \frac{-2T_s}{r} \cos \alpha \quad (48-1b) \]

For water in a clean glass tube, \( \alpha = 0 \) and \( \rho = r \).
Figure 49-1  Average capillary heights and tensions in soils.

<table>
<thead>
<tr>
<th>Soil</th>
<th>Height (m)</th>
<th>Tension (kN/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sand</td>
<td>0.05–1</td>
<td>0.5–10</td>
</tr>
<tr>
<td>Silt</td>
<td>1–10</td>
<td>10–100</td>
</tr>
<tr>
<td>Clay</td>
<td>&gt;10</td>
<td>&gt;100</td>
</tr>
<tr>
<td>Maximum</td>
<td>&gt;35</td>
<td>&gt;350</td>
</tr>
</tbody>
</table>

is held in place by capillary attraction and exerts relatively large stabilizing forces on the structure of the soil.

In fine-grained soils, the capillary zones reach to a considerable height above the water table (Fig. 49-1); in coarse sand, the capillary rise may be negligible. The capillary rise in soil is similar to capillary rise in a capillary tube, as shown in Fig. 48-1. The forces holding the column of water in the tube in equilibrium are complex, but an accurate expression for the height of rise in the tube may be obtained by assuming that the water surface acts as a membrane which may act in tension to support the weight of the column of water.

The pressure distribution in the water column of Fig. 48-1 is triangular in shape. The pressure is equal to zero at point A and $-\gamma_c h$ at point B. The curvature of the air-water interface is $1/\rho$, and the pressure difference across the interface is related to this curvature, as shown in Eq. 48-1b. In natural soil masses, as the height above the water table increases, the water content decreases, the curvature of the air-water interface increases, and the pressure difference across the interface increases. Negative pressures in the water film in soils with very low moisture content may become very large (Fig. 49-1). The application of Eqs. 48-1a and 48-1b is illustrated in the example problem of Fig. 49-2.

Figure 49-2

**Example Problem**  For the capillary tube of Fig. 48-1, determine the capillary height $h$ and the negative pressure $u_B$ at the interface for a 0.1-mm radius glass tube in water.

From Eq. (48-1a),

$$h = \frac{2(0.0735 \text{ N/m})}{(9810 \text{ N/m}^3)(0.0001 \text{ m})} = 0.150 \text{ m}$$

From Eq. 48-1b,

$$u_B = -\frac{2(0.0735 \text{ N/m})}{0.0001 \text{ m}} = -1.47 \text{ kN/m}^2$$
SATURATED FLUID FLOW

Darcy's Law

Water both below the water table and in the capillary zone is subject to forces that cause flow. In the zone below the water table, changes in pressure and elevation are the most important causes of flow. The property of soil that permits the passage of water under a gradient of force is called permeability.

In saturated soil, unless pores are very large, water flow is laminar. Reynolds (1883) has discussed a critical condition for water flow in pipes that describes a division between laminar and turbulent flow at a Reynolds number \( \nu D p/\mu = 2100 \). In soils a critical Reynolds number also exists, but is approximately in the range from 1 to 10.

For flow in the laminar range, energy losses are proportional to the first power of velocity, and Darcy (1856) has suggested Eq. 50-1a for velocity.

The head loss \( (h_L) \) in this equation is the name given to the energy loss per unit weight of fluid flowing and may be represented by a decrease in the Bernoulli

Figure 50-1

Darcy's equation for velocity and flow rate is

\[
v = -\frac{h_L}{L} = -ki
\]  
(50-1a)

\[
q = \nu_nA_n = \nu A
\]  
(50-1b)

where

- \( q \) = Volume flow rate
- \( h_L \) = Energy loss per unit weight (head loss)
- \( \nu \) = Darcy velocity (superficial velocity)
- \( \nu_n \) = Actual velocity
- \( A_n \) = Area of voids in cross section
- \( A \) = Area of total cross section
- \( L \) = Length of flow

\[ i = \text{Hydraulic gradient} \left( \frac{h_L}{L} \right) \]

\( k \) = Coefficient of permeability

(Note: The negative sign in Darcy's equation indicates that the velocity is in the direction of the negative gradient. The equation is often written without the negative sign.)
head terms, which are (in units of length)

\[ \text{velocity head } = h_v = \frac{v^2}{2g} \]
\[ \text{pressure head } = h_p = \frac{u}{\gamma_w} \]
\[ \text{elevation head } = h_e = \gamma v \]

where \( u \) is water pressure (pore pressure). The difference between the total Bernoulli heads at two points on the same flow line is the magnitude of the head loss \( h_L \). The total Bernoulli head is equal to the sum of the head terms.

\[ h_T = h_p + h_e + h_v \]

and

\[ h_{T1} = h_{T2} + h_L \]

The head loss represents the amount of energy lost in heat through the viscous action between layers of flowing water.

It is more convenient in soil flow problems to use the total cross-sectional area as the area of flow than to find the area of voids [Eq. 50-1b]. Therefore, the flow \( q \) is usually represented by the equation \( q = vA \), where \( A \) is the total cross-sectional area, including voids and solids, and \( v \) is a superficial velocity (also called Darcy's velocity) that would exist if the total section were a

---

**Figure 51-1**

**Example Problem** Water flows through a soil mass that has a length of 4 m and a cross-sectional area (\( A \)) of 2 m\(^2\). The fluid energy lost when 1 m\(^3\) of water flows through the soil is 1500 N m. The void ratio of the soil is 0.64. The elapsed time for this flow is 30 hr. Find \( v \), \( v_n \), and \( k \). From Eq. 50-1,

\[ v = \frac{q}{A} = \frac{1}{30(3600)2} = 4.63 \times 10^{-6} \text{ m/sec} \]

\[ v_n = \frac{v}{n} = v(1 + e)/\varepsilon = 1.19 \times 10^{-5} \text{ m/sec} \]

\[ h_L = \frac{\text{energy loss}}{\text{unit weight}} = \frac{1500 \text{ N m}}{1 \text{ m}^3(9810) \text{ N/m}^3} = 0.15 \text{ m} \]

\[ k = \frac{v}{i} = \frac{vL}{h_L} = \frac{4.63 \times 10^{-6} \times 4}{0.15} \]

\[ k = 1.23 \times 10^{-4} \text{ m/sec} \]
flow section. This equation is called Darcy’s equation, and the term velocity, unless otherwise noted in this discussion, refers to superficial velocity.

In other disciplines the coefficient \( k \), or constant of proportionality between velocity and hydraulic gradient, is termed a coefficient of conductivity, since it depends upon fluid properties as well as properties of the soil medium. However, through long-term use in geotechnical literature, \( k \) is popularly called the coefficient of permeability. Such usage is probably justified, since the temperature of water varies little in most soil applications. (The kinematic viscosity of water decreases about 23% from 10° to 20°C.) Thus, in coarse-grained soils the primary factors affecting \( k \) are the soil particle size and gradation, particle shape and roughness, and the void ratio of the soil medium. In fine-grained soils, where particle surface forces predominate, other factors, such as the type of clay mineral and adsorbed ions, have an overriding influence on \( k \). The application of Darcy’s law is illustrated in the example problem of Fig. 51-1.

**Hydraulic Gradient**

The hydraulic gradient used in Darcy’s equation, \( i = h/L \), may be found quite easily in simple flow situations in the laboratory, in which reservoirs of water occur on each side of the soil sample. One such situation is shown in Fig. 53-1. The total head will be taken equal to the pressure head plus the elevation head. The velocity head in most soil flow situations is negligible compared to the pressure and elevation heads and is generally neglected.

The example in Fig. 53-1 gives the values of elevation head, pressure head, and total head at different points in the apparatus. The values of head at \( b \) and \( d \) may be found, after picking an arbitrary elevation datum, by adding the distance above the elevation datum and the distance below the surface of the water. Point \( d \) has a pressure head that may be found by using the principle that the pressure at equal elevations in static bodies of fluid is equal; therefore, the pressure at point \( e \) and the pressure at \( d \) are equal. It is true that the fluid in the tube is not exactly at rest due to the small amount of flow through the soil sample; however, these quantities of flow are small enough so that the head loss along the edge of the container in the tube is negligible, and the total head at any point in the tube may be considered constant.

The total head at point \( c \) in the middle of the soil sample may be found by examining the gradient of head through the sample. Since the sample is saturated and homogeneous and since the equation of continuity requires a constant velocity through the soil sample, the head loss per unit distance must also be constant; therefore, the head at point \( c \) is equal to the average of the heads at points \( b \) and \( d \). The pressure head at \( c \) may then be found by using the relationship \( h_{tr} = h_{pr} + h_{re} \).
**Example Problem**  Find the pressure head at point c by completing the following table.

<table>
<thead>
<tr>
<th>Point</th>
<th>Pressure Head ($h_p$)</th>
<th>Elevation Head ($h_e$)</th>
<th>Total Head ($h_T$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>0</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>b</td>
<td>2</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>c</td>
<td>(5)</td>
<td>1</td>
<td>(6)</td>
</tr>
<tr>
<td>d</td>
<td>8</td>
<td>0</td>
<td>8</td>
</tr>
</tbody>
</table>

**Determination of Permeability**

The coefficient of permeability may be found by field tests or laboratory tests. If laboratory tests are used and the soil is fine grained, undisturbed samples of the soil must be taken to determine the in-situ values of $k$. For a sandy soil, $k$ can be obtained accurately by remolding to the proper void ratio. Figure 54-1 shows a
Darcy's law for Flow in a Constant Head Permeameter
\[ q = \frac{Q}{t} = vA = kiA = \frac{kh_L A}{L} \]

from which,
\[ k = \frac{QL}{th_L A} \] (54-1)

where
- \( Q \) = flow volume in time \( t \)
- \( A \) = area of soil sample (cross section)
- \( L \) = length of soil sample
- \( h_L = H \)

sample that has been placed into a constant head permeability apparatus. In this figure the water flow is upward. The coefficient of permeability may be found by applying Darcy's law to the flow (Eq. 54-1). From this equation, \( k \) may be computed by measuring \( Q \) for a given time interval. An example problem utilizing the data from a constant head permeability test is worked in Fig. 55-1.

Figure 55-2 shows a sample in a variable head permeability apparatus. The water level in the tube at the beginning of the test is \( H_1 \) and at the end of the test, \( H_2 \). At some intermediate time, the level is \( H \) and the change in level during a
Figure 55-1

Example Problem  Given the following data from a constant head permeability test, compute the coefficient of permeability $k$.

\[ Q = 0.034 \text{ m}^3 \]
\[ t = 500 \text{ sec} \]
\[ H = 2.0 \text{ m} \]
\[ L = 0.20 \text{ m} \]
\[ A = 0.04 \text{ m}^2 \]

From Darcy's law,
\[ k = \frac{Q}{t} \frac{L}{h_t A} = \frac{(0.034 \text{ m}^3)(0.20 \text{ m})}{(500 \text{ sec})(2 \text{ m})(0.04 \text{ m}^2)} = 1.7 \times 10^{-4} \text{ m/sec} \]

Figure 55-2  Variable head permeameter.

Darcy's Equation
\[ -a \, dH = dQ = kA \, \frac{H}{L} \, dt; \quad \int_{H_1}^{H_2} \frac{dH}{H} = \int_{0}^{t} \frac{kA}{La} \, dt; \quad k = \frac{aL}{At} \ln \left( \frac{H_1}{H_2} \right) \]

where
\[ a = \text{area of tube} \]
\[ A = \text{area of soil sample (cross section)} \]
\[ dQ = \text{flow volume in time } dt \]
Figure 56-1  Permeability measured by the auger hole method.

\[ -dy(\pi a^2) = k(2\pi a H) \frac{y}{L} \, dt + k\pi a^2 \frac{y}{L} \, dt \]

\[ k = \frac{aL}{(2H + a) t} \ln \frac{y_1}{y_2} \]

\( L = \) An empirical length over which the head loss \( y \) occurs.
\( L = aH/0.19 \) (meters)

**Hooghoudt Equation**

**Differential equation:**

**Ernst Equation**

\[ k = \frac{40}{\left(20 + \frac{H}{a}\right) \left(2 - \frac{y}{H}\right)} \frac{a}{\Delta y} \] (lengths in meters)

\( \Delta y = \) rise in water level during time \( \Delta t \).
\( y = \) average \( y \) during increment.

small time interval, \( dt \), is \( dH \). The permeability \( k \) may again be computed by applying Darcy’s equation as shown (water level, \( H \), is equal to head loss, \( h_L \)).

A field method for a preliminary estimate of the coefficient of permeability is the single auger hole method. An auger hole is drilled in the field and the water
is removed by bailing. Hooghoudt (1936) investigated the case of an auger hole in a homogeneous soil and assumed that the water table remained in a horizontal position and that water flowed horizontally into the sides of the auger hole and vertically through the bottom of the hole (see Fig. 56-1). The derivation is similar to a variable head laboratory test. Hooghoudt's differential equation for flow has an empirical constant, $L$, which represents some length over which the head loss, $y$, occurs (see Fig. 56-1). Experimentally, he found that $L$ equals $aH/0.19$. His equation may be integrated as shown. Ernst's (1950) revised estimate of $k$ from this same test is also shown in Fig. 56-1. The Hooghoudt and Ernst equations are illustrated in the example of Fig. 57-1.

Another field estimate of the coefficient of permeability may be obtained from wells by a pumping test. A steady-state flow is established and $k$ is found

---

**Figure 57-1**

**Example Problem**  A 3 m deep auger hole (diameter = 0.2 m) is drilled in a homogeneous soil. The hole extends 2.5 m below the water table. Readings on the water table are as follows:

<table>
<thead>
<tr>
<th>Time (sec)</th>
<th>$y$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2.5</td>
</tr>
<tr>
<td>600</td>
<td>1.6</td>
</tr>
<tr>
<td>1200</td>
<td>1.0</td>
</tr>
<tr>
<td>1800</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Find the coefficient of permeability ($k$)

**Hooghoudt Equation**

$$k = \frac{aL}{(2H + a)t} \ln \frac{y_1}{y_2} = \frac{(0.1 \text{ m})}{\frac{(0.1 \text{ m} \times 2.5 \text{ m})}{0.19 \text{ m}}} \ln \frac{2.5}{0.5}$$

$$k = 2.3 \times 10^{-5} \text{ m/sec}$$

**Ernst Equation** (use interval 600 to 1200 sec)

$$k = \frac{40}{\left(20 + \frac{2.5}{0.1}\right) \left(2 - \frac{1.3}{2.5}\right) \left(\frac{0.6 \text{ m}}{600 \text{ sec}}\right)}$$

$$k = 4.62 \times 10^{-5} \text{ m/sec}$$

---

Saturated Fluid Flow
from Eq. 58-1. Field measurements are made to determine \( h \) at two or more distances \( r \) from the well centerline.

In the absence of a measured permeability, the coefficient for clean granular soils can be estimated from the following equation.

\[
k = \frac{2g \rho}{C_s \mu} D^2 \frac{e^3}{1 + e}
\]  

(58-2)

where \( g \) is the acceleration of gravity; \( \mu/\rho \) is the kinematic viscosity of water; \( C_s \) is a particle shape factor, which varies from 360 for spherical particles to about 700 for angular particles; \( D \) is a weighted or characteristic particle diameter; and \( e \) is the void ratio. The characteristic diameter \( D \) is obtained from a grain-size analysis using the following equation.

\[
D = \frac{\Sigma M_i}{\Sigma (M_i/D_i)}
\]

where \( M_i \) is the mass retained between two adjacent sieves and \( D_i \) is the mean diameter of the adjacent sieves. (The kinematic viscosity of water varies approximately linearly from 1.31 mm²/sec at 10°C to 1.01 mm²/sec at 20°C.) The use of Eq. 58-2 is illustrated in the example problem of Fig. 59-1.
Figure 59-1

Example Problem Estimate the permeability for sand of the gradation shown when the dry unit weight is 14.2 kN/m$^3$ and the grain shape is subangular. The water temperature is 15°C. Assume $G = 2.65$.

<table>
<thead>
<tr>
<th>Sieve no.</th>
<th>Sieve Size (mm)</th>
<th>Mean Sieve Size $(D_i - mm)$</th>
<th>Mass Retained $(M_i)$</th>
<th>$M_i/D_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>2.00</td>
<td>1.43</td>
<td>0.5</td>
<td>0.4</td>
</tr>
<tr>
<td>20</td>
<td>0.85</td>
<td>0.73</td>
<td>5.2</td>
<td>7.1</td>
</tr>
<tr>
<td>30</td>
<td>0.60</td>
<td>0.51</td>
<td>30.6</td>
<td>60.0</td>
</tr>
<tr>
<td>40</td>
<td>0.425</td>
<td>0.34</td>
<td>45.4</td>
<td>133.5</td>
</tr>
<tr>
<td>60</td>
<td>0.25</td>
<td>0.22</td>
<td>13.2</td>
<td>60.0</td>
</tr>
<tr>
<td>80</td>
<td>0.180</td>
<td>0.16</td>
<td>5.1</td>
<td>31.9</td>
</tr>
<tr>
<td>100</td>
<td>0.150</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**TOTA LS** 100.0 292.9

$$D_m = \frac{\Sigma M_i}{\Sigma M_i/D_i} = \frac{100}{293} = 0.34 \text{ mm} = 340 \mu \text{m}$$

$$e = G \frac{\gamma_w}{\gamma_d} - 1 = 2.65 \frac{9.81}{14.2} - 1 = 0.83$$

The kinematic viscosity of water is

$$\mu/\rho = 1.16 \text{ mm}^2/\text{sec} \text{ at } 15^\circ\text{C}.$$  

Estimate $C_s = 600$.

$$k = \frac{2g}{C_s} \frac{D^2 \rho e^3}{\mu \left(1 + e\right)} = \frac{2(9.81 \text{ m/sec}^2)(340 \times 10^{-6} \text{ m})^2}{600(1.16 \times 10^{-5} \text{ m}^2/\text{sec})} \frac{0.83^3}{1.83}$$

$$k = 1.0 \text{ mm/sec}$$

**Soil Permeability**

Some representative values of the coefficient of permeability for different soils are shown in Fig. 60-1. Note the extreme variation in permeability from gravel to clay.
Figure 60-1 Typical values of permeability.

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>Coefficient of Permeability (Range in mm/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gravel</td>
<td>$&gt;10$</td>
</tr>
<tr>
<td>Sand</td>
<td>$10^{-4}$</td>
</tr>
<tr>
<td>Silt</td>
<td>$10^{-4}$ - $10^{-6}$</td>
</tr>
<tr>
<td>Clay</td>
<td>$&lt;10^{-6}$</td>
</tr>
</tbody>
</table>

Permeability of Nonhomogeneous Soils

In some cases such as that of the stratified bed of soil in Fig. 60-2, the proper value of the coefficient of permeability for the soil may not be found by taking the numerical average of the coefficient of permeability at several different locations. The coefficient of permeability to be used for flow through this type of soil deposit depends on the direction of flow. In some instances it is possible to compute effective coefficients of permeability. Development of effective permeability in stratified soils for horizontal flow and vertical flow are shown in Figs. 60-2 and 61-1.

Figure 60-2 Coefficient of permeability for horizontal flow through stratified nonisotropic soil.

Horizontal Flow

\[ q = q_1 + q_2 + q_3 \]
\[ k_x i \Sigma H_j = k_{x_1} i H_1 + k_{x_2} i H_2 + k_{x_3} i H_3 = \Sigma [k_x i H_j] \]
\[ k_x i \Sigma H_j = i \Sigma (k_i H_j) \]

so

\[ k_x = \frac{\Sigma (k_i H_j)}{\Sigma H_j} \]  \hspace{1cm} (60-2)
One-Dimensional Flow

Horizontal Flow (Fig. 60-2)
Assume a case in which the total head along line AB has a constant value, which would occur in hydrostatic pressure. Also, along line CD, a constant head exists, but at a value lower than along line AB. Flow will then be horizontal in each of the three layers shown. The effective coefficient of permeability in the x direction is, therefore, equal to the weighted average of the values of $k_x$ in the three layers (Eq. 60-2).

Vertical Flow (Fig. 61-1)
If the gradient in head is in the vertical direction, then Eq. 61-1 may be used to estimate an effective coefficient of permeability through the horizontally stratified soil.

---

**Figure 61-1** Coefficient of permeability for vertical flow through stratified, non-isotropic soil.

![Diagram](image)

**Vertical flow**

- $q = q_1 = q_2 = q_3$
- $\Sigma(h_L)_i k_y A = k_{y1} \frac{h_{L1}}{H_1} A = k_{y2} \frac{h_{L2}}{H_2} A = k_{y3} \frac{h_{L3}}{H_3} A$
- $\Sigma(h_L) = \frac{q \Sigma H_j}{k_y A} = \frac{q H_1}{k_{y1} A} + \frac{q H_2}{k_{y2} A} + \frac{q H_3}{k_{y3} A}$
- $\Sigma \frac{H_j}{k_y} = \left[ \Sigma \frac{(H)}{k_y} \right]_{j}$
- and
- $k_y = \frac{\Sigma \frac{H_j}{k_y}}{\Sigma \left[ \frac{(H)}{k_y} \right]_{j}}$ (61-1)
Figure 62-1

**Example Problem** The three layers of soil shown in Fig. 61-1 represent the soil profile beneath a reservoir in which the water depth is 10 m. Other data are as follows.

<table>
<thead>
<tr>
<th>Layer</th>
<th>( H ) (m)</th>
<th>( k_x ) ( \text{\frac{mm}{sec}} )</th>
<th>( k_y ) ( \text{\frac{mm}{sec}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>( 2.0 \times 10^{-4} )</td>
<td>( 3.0 \times 10^{-5} )</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>( 1.5 \times 10^{-5} )</td>
<td>( 3.0 \times 10^{-6} )</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>( 1.0 \times 10^{-5} )</td>
<td>( 1.0 \times 10^{-6} )</td>
</tr>
</tbody>
</table>

A sandy layer lies below this profile. The sand has horizontal drainage, and the pore pressure in the sand is essentially zero.

Assume vertical flow through the layers shown and compute the water loss in three months for the reservoir \((A = 3000 \text{ m}^2)\).

\[
k_y = \frac{6}{\left( \frac{2}{3.0 \times 10^{-5}} + \frac{1}{3.0 \times 10^{-6}} + \frac{3}{1.0 \times 10^{-6}} \right)}
\]

\[
= 1.76 \times 10^{-6} \text{ mm/sec}
\]

\[
Q = k_y \cdot j \cdot A \cdot t = \left[ \left( \frac{1.76 \times 10^{-6}}{1000} \text{ m/sec} \right) \left( \frac{16 \text{ m}}{6 \text{ m}} \right) \left( 3000 \text{ m}^2 \right) \right] = 109 \text{ m}^3
\]

Although the above effective values of coefficient of permeability may be used in flow situations where the direction of flow is horizontal or vertical (one dimensional), if the flow is curved it may be impossible to reduce this stratified soil to a simple homogeneous anisotropic soil mass by the method shown above. However, the example problem of Fig. 62-1 serves as an example of what may be done when the flow direction is known and the proper data determining permeability of the soil layers can be obtained.

**Two-dimensional Flow — Flow Nets**

Figure 63-1 shows the flow net for flow through the constant head permeameter in Fig. 54-1. The vertical lines in the soil sample represent the direction of flow.
of water particles and are called flow lines. As the water progresses through the sample, head is lost at a constant rate and the horizontal lines represent lines of constant head. The two mutually perpendicular families of curves, flow lines and constant head lines, form a flow net. The flow net is a useful device in flow problems, because, once it is found, the values of the head at any point in the soil sample are uniquely determined.

The flow net of Fig. 63-2 has curved flow lines. This relatively simple laboratory experiment consists of an impermeable rectangular tank filled with
Figure 64-1 Derivation of the Laplace equation.

\[ v_y + \frac{\partial v_y}{\partial y} \, dy \]

\[ v_x + \frac{\partial v_x}{\partial x} \, dx \]

An element of soil in two-dimensional curved flow field.

**Conservation of Mass**

\[ v_x \, dy \, dz - \left( v_x + \frac{\partial v_x}{\partial x} \right) dy \, dz + v_y \, dx \, dz - \left( v_y + \frac{\partial v_y}{\partial y} \right) dx \, dz = 0 \]

where \( dz \) is the dimension of the element perpendicular to the two-dimensional flow.

From the preceding equation,

\[ \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0 \]

Now, since

\[ v_x = -k_x j_x = -k_x \frac{\partial h}{\partial x} \quad \text{and} \quad v_y = -k_y \frac{\partial h}{\partial y} \]

then

\[ k_x \frac{\partial^2 h}{\partial x^2} + k_y \frac{\partial^2 h}{\partial y^2} = 0 \]  \hspace{1cm} (64-1a)

For problems in which the coefficient of permeability in both directions is the same (isotropic), \( k \) may be cancelled and the equation becomes the Laplace equation

\[ \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0 \]  \hspace{1cm} (64-1b)

The equation is sometimes written

\[ \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \]  \hspace{1cm} (64-1c)

where \( \phi = kh \) and is called the velocity potential.
soil that has a cutoff wall extending halfway through the soil. By maintaining a higher water level on the left-hand side of the cutoff wall, flow occurs from left to right. We may analyze the conditions of the flow at the element shown to determine the characteristics of curved flow. An enlarged picture of this element is shown in Fig. 64-1. Flow entering from the left and bottom of the element has velocities of $v_x$ and $v_y$, respectively. The flow leaving the top of the element has a velocity equal to the velocity entering, plus the change in velocity in the distance $dy$. A similar incremental velocity has been added to $v_y$ on the right-hand side of the sample. Conservation of mass specifies that if the volume of the sample remains the same and if the sample is completely saturated, then the net flow will equal zero. The derivation in Fig. 64-1 is based on this law. Either Eq. 64-1b or 64-1c is known as the LaPlace equation and describes the distribution of head in the interior of the homogeneous, isotropic soil region.

The solution to LaPlace's equation may be found by several different methods. Some of these are:

1. Direct mathematical solution.
3. Electrical analogy solution.
4. Graphical solution.

**Direct Mathematical Solution**

A demonstration of mathematical solutions can be made by picking functions that satisfy LaPlace's equation and choosing the boundary conditions that make the solution apply to a physical problem. Some examples of this method of solution follow.

**Example 1**
Consider the equation $h = 4y$, which satisfies LaPlace's equation. Values of $h$ are plotted on the coordinate system in Fig. 66-1a. In Fig. 66-1b boundaries have been placed around a portion of the flow field to indicate a physical problem of which this equation is a solution. The values of $h$ in Fig. 66-1b satisfy not only LaPlace's equation, but also the boundary conditions and therefore represent the proper solution.

**Example 2**
Consider the equation $h = 2xy$. This equation also satisfies LaPlace's equation. Values of $h$ are plotted on the coordinate system in Fig. 67-1, and boundaries are placed in the flow field to show a physical problem.

**Example 3**
$h = C_1 \log C_2(x^2 + y^2)^{1/2}$. This equation satisfies LaPlace and is a solution for the problem shown in Fig. 68-1, in which water is being pumped from a well and is
Figure 66-1  Application of the LaPlace equation to one-dimensional flow.

(a) Head distribution of $h = 4y$

(b) Physical boundaries for $h = 4y$
flowing through a permeable aquifer bounded on the top and bottom by relatively impermeable soil.

The type of flows considered in this section are limited to two-dimensional flows. This theory may be extended to three-dimensional flow by including the direction perpendicular to the paper in the derivation of the differential equation. It is apparent that any number of possible solutions to problems may be analyzed, with the intent of working from the possible solution to the physical problem it represents. However, this method is not particularly suited for solving a problem with difficult boundary conditions. A more proper mathematical method is available using the theory of complex variables, but this method will not be discussed here. Numerical solutions to flow nets are discussed in Chapter 10.
Figure 68-1  Application of the LaPlace equation to axisymmetric flow.

Physical problem for

\[ h = C_1 \log C_2 (x^2 + y^2)^{1/2} \]

Well flow in confined aquifer

**Electrical Analogy Solution**

A method of obtaining a solution to a two-dimensional flow problem using an electrical flow field analogy has proved very useful in some difficult flow problems. This method is based on the fact that steady-state electrical flow through a conductor of uniform thickness may also be represented by the LaPlace equation. For example, in Fig. 69-1 the flow field actually represents the flow through a permeable soil under a concrete dam. However, this problem may be solved by using a homogeneous electrical conducting medium in the shape of the flow field. (Conducting paper is often used.) Silver electrodes are then attached or painted on the medium along lines \( AB \) and \( CD \). A difference in electrical potential is applied to these two electrodes, and the electrical flow occurs along the flow lines shown. The position of the constant head lines may be found by tracing lines of constant electrical potential (voltage). Flow lines may then be drawn perpendicular to the constant head lines. The flow net found in this manner is the same flow net that will exist in a hydraulic flow field.
Graphical Solution
Flow nets such as those shown in Figs. 63-2 and 69-1 consist of two families of mutually perpendicular curves. These two families of curves represent the distribution of head within a flow field (constant head lines) and the path followed by water flowing through the field (flow lines). When the constant head lines are properly drawn, they describe the unique solution to the Laplace equation (Eq. 64-1b) for the specific boundary conditions of the problem. A convenient way to obtain the flow net is by trial-and-error sketching of the flow lines and constant head lines. Several simple criteria must be satisfied in sketching the flow net.

1. The flow lines and constant head lines must be mutually perpendicular.
2. The areas formed by the intersection of the flow lines and constant head lines should form "square" figures.
3. The flow net must satisfy the boundary conditions of the flow field.

The criteria of square figures is developed in Fig. 70-1. The figure also demonstrates that the head drop across each square is constant and that the flow in any flow path equals the flow in any other flow path.

The graphical method for finding flow nets is most easily completed as follows.

Refer to Fig. 69-1. On a sheet of unlined paper draw to scale in ink the boundaries of the flow field, and then, using a soft-lead pencil, sketch four or five
**Figure 70-1** Derivation of flow rate through two-dimensional flow field.

Consider the flow through squares a and c in Fig. 69-1. Since these squares are in the same flow path and since the flow is a steady flow, the value of \( q \) through the two squares will be the same. Therefore, from Darcy’s law,

\[ q_a = k \frac{\Delta h_a}{L_a} B_a = q_c = k \frac{\Delta h_c}{L_c} B_c \]

If the flow net is sketched so that \( B/L \) is constant, the head drop across each square will be a constant. The most convenient ratio to maintain is \( B/L \) equal to 1. This produces “square” figures.

Also

\[ q_a = k \Delta h_a, \quad q_b = k \Delta h_b \]

\( \Delta h_a = \Delta h_b \), because these two squares lie between the same equal head lines. Therefore, \( q_a = q_b \), so the flow in any flow path equals the flow in any other path. Now,

\[ q = q_1 + q_2 + \ldots + q_m = q_1 M = k \Delta h_a M \]

\[ q = k h_L \frac{M}{N} \]

(70-1)

where \( M \) is the number of flow paths, \( N \) is the number of equal head drops, and \( h_L \) is the total head loss through the net.

In the case of partial squares, either \( N \) or \( M \) may include a fractional square.

possible positions for flow lines. The first approximation for these lines should have a fairly uniform spacing. Then sketch constant head lines, making an attempt to keep all figures square and all intersections with flow lines at right angles. For the purpose of this method, a square figure may be defined as one in which the median flow line and the median equal head line through the center of the figure have the same length and that has 90° corners at all intersections. After drawing the complete family of constant head lines, erase all the flow lines and redraw them, attempting to keep the figures square and the intersections perpendicular. Then erase the constant head lines and redraw them. This procedure may be repeated until the figures all satisfy the conditions given. The flow net arrived at in this manner is the same flow net that would be produced by a mathematical solution if one were available for this problem because there is only one pair of mutually perpendicular families of curves that will solve this problem, and that pair may be found by any of the methods suggested here.
During the sketching of the flow net, it may become obvious that an entire set of figures such as the shaded area of Fig. 69-1 cannot be drawn as squares but will turn out to be some fraction of a square. This will still be a proper flow net if the ratio of length to width of all the figures in such a line is constant. After the sketching procedure is completed, the flow through the net may be found, as shown in Fig. 70-1.

The water pressure at any point such as s in the flow net in Fig. 69-1 may be found from the flow net.

\[(h_t)_s = (h_e + h_p)_s = \left( h_e + \frac{u}{\gamma_w} \right)_s \]

The total head \((h_T)_s\) is equal to

\[h_{AB} = \frac{9}{14.3} h_L.\]

and

\[u_s = \gamma_w(h_T - h_e)_s\]

Velocities in the flow net may be found, for example, at point s by taking the drop in head between r and t, dividing by the distance, and then applying Darcy’s law.

\[v = -\frac{k\Delta h_{r-t}}{rt}\]

**Two-dimensional Flow — Anisotropic Soil**

A modification of the above methods is necessary in soils in which the horizontal and vertical coefficients of permeability are different. The permeabilities \(k_x\) and \(k_y\) then remain in Eq. 64-1a. Because this equation is not LaPlace’s equation, the method of solution will not be the same. Consider a case in which the horizontal coefficient of permeability is greater than the vertical coefficient, as shown in Fig. 72-1. Let the area shown represent a mass of soil in which flow is occurring. The flow net in this situation will not be composed of mutually perpendicular families of curves. However, if we contract the \(x\) dimension, keeping the total resistance to flow in the \(x\) direction a constant, the resistance per unit distance increases and we may form a soil mass that has equal coefficients of permeability in both directions (transformed section). Let \(x' = ax\) be the new dimension on the transformed section; then the LaPlace equation holds for the transformed section (Fig. 72-1) and the flow net in this section may be found in the usual manner. Intersections between flow lines and equal head lines may then be transformed back to the real section by multiplying the \(x'\) dimension by the factor \(1/a\). In order to find the flow through the flow net in the transformed section in Fig. 72-1b; we may use the equation

\[q = k'h_L \frac{M}{N}\]
(a) Actual flow net for $k_x > k_y$

(b) Flow net—transformed section

Develop the transformation factor $a$.

If $x' = ax$,\[ \frac{\partial h}{\partial x'} = \frac{\partial h}{\partial x} \frac{\partial x}{\partial x'} + \frac{\partial h}{\partial y} \frac{\partial y}{\partial x'} \]

but\[ \frac{\partial y}{\partial x'} = 0 \]
\[
\frac{\partial h}{\partial x'} = \frac{\partial h}{\partial x} \frac{\partial x}{\partial x'} = \frac{\partial h}{\partial x} \frac{1}{a} \quad ; \quad \frac{\partial^2 h}{\partial x'^2} = \frac{\partial (\partial h/\partial x')}{\partial x} \frac{\partial x}{\partial x'} = \frac{\partial^2 h}{\partial x^2} \frac{1}{a} \frac{1}{a}
\]

Substituting into Eq. 64-1a,

\[
k_x a^2 \frac{\partial^2 h}{\partial x'^2} + k_y \frac{\partial^2 h}{\partial y^2} = 0 \quad (73-1)
\]

which becomes the Laplace equation if

\[
a^2 = \frac{k_y}{k_x}
\]

\[
a = \sqrt{\frac{k_y}{k_x}}
\]

(M is number of flow paths and N is number of head drops.) However, the value of the coefficient of permeability \(k'\) will be different from either \(k_x\) or \(k_y\). It may be found by analyzing the flow through two squares, one taken from the flow net of the real section and the corresponding one from the flow net of the transformed section (Fig. 74-1).

The example problem of Fig. 75-1 illustrates the computation of flow under an impermeable dam on an anisotropic soil deposit.

**Flow in Earth Dams**

In many soil problems involving flow nets, the boundaries of the flow net may not all be specified as in previous examples. One such case is in the flow through a homogeneous earth dam. The top flow line is a free surface and its exact position is not known. In order to approximate the position of this flow line, assume a dam having an upstream face in the shape of a parabola (Fig. 75-2) and an underdrain extending to a point directly under the intersection of the water surface and the upstream face of the dam. This is not a common design. However, if the dam were constructed in this manner and if the parabola had the focus at point \(F\), the mutually perpendicular lines forming the flow net would be parabolas and the flow net would be as shown.

Changing the upstream face to a straight line and changing the position of the drain affects the position of the top flow line very little, and a parabola may generally still be used as an approximation to the top flow line (Fig. 76-1). This parabola is called the basic parabola. Studies of experimental flow nets have shown that the basic parabola intersects the water surface at a point \(A\) where \(AB\) is approximately 0.3 GB (Casagrande, 1937). The vertical line at \(E\) is called the
Figure 74-1  Flow rate through anisotropic soil. These squares are $a$ and $d$ in Fig. 72-1.

Square (a)—Real section

Square (d)—Transformed section

$$q_a = k_x \frac{\Delta h_a}{L} b = q_d = k' \frac{\Delta h_d}{L'} b;$$

$$k' = k_x \frac{L'}{L} = k_x a = \sqrt{k_x k_y}$$  \hspace{1cm} (74-1a)

and

$$q = q' = k'h_l \frac{M}{N}$$  \hspace{1cm} (74-1b)
Figure 75-1
Example Problem  Use the dam shown in Fig. 72-1 (head upstream = 10 m, downstream = 0 m) $k_x = 2 \times 10^{-5}$ mm/sec, $k_y = 5 \times 10^{-6}$ mm/sec. Find the rate of flow beneath the dam.

$$a^2 = \frac{k_y}{k_x} = \frac{5 \times 10^{-6}}{2 \times 10^{-5}} = \frac{1}{4}$$

Note. The transformed section shown in Fig. 72-1 was drawn for this ratio.

$$k' = \sqrt{k_x k_y} = 1 \times 10^{-5} \text{ mm/sec}$$

$$q = k' h L \frac{M}{N} = \left( \frac{1 \times 10^{-5}}{1000} \text{ m/sec} \right) (10 \text{ m}) \left( \frac{4}{8} \right)$$

$$q = 5 \times 10^{-8} (\text{m}^3/\text{sec})/\text{m}$$

Figure 75-2  Flow net with a free surface.

Flow net—confocal parabolas
directrix of the parabola, and point A is equidistant from E and F. Therefore,

\[ \sqrt{H^2 + L^2} = L + S \]

where \( S = \overline{FJ} \). \( S \) may be found from this equation and the directrix located. Points on the basic parabola may be found by using the geometric fact that each parabolic point is equidistant from the focus and the directrix; therefore,

\[ \sqrt{x^2 + y^2} = x + S \]

The top flow line may be plotted from this equation, and the flow net sketched by the usual method (Fig. 77-1).

Since the parabola representing the top flow line does not intersect the upstream face of the dam at the water level, it is necessary to make a correction in the upper part of the top flow line. This correction is shown in Fig. 77-1, where the corrected top flow line meets the upstream face (which is an equal head line) at a 90° angle. In sketching the flow net, it is important to know that the intersections of equal head lines with the top flow line are equally spaced in the vertical direction, since head losses along this line are entirely losses in elevation head. The total flow through the net may be found by Eq. 70-1.

The total flow through the net may also be found by examining the symmetrical lower section of the net (see Fig. 77-1b). Note that \( M = N \) in this section. Point \( R \), directly above the focus, is equidistant from the focus and the directrix. The total head at point \( R \) is equal to \( S \) (so \( h_L = S \)), and \( \overline{RT} \) is a constant head line; therefore

\[ q = kh_L \frac{M}{N} = kS \]
Figure 77-1

(a) Flow net for earth dam

(b) Confocal parabolas

---

Figure 77-2

Example Problem  For the dam of Fig. 77-1, \( k = 2 \times 10^{-5} \text{ mm/sec} \), find the top flow line and draw the flow net. Find the rate of flow.

\[
S = \sqrt{H^2 + L^2} - L = \sqrt{15.0^2 + 19.3^2} - 19.3 = 5.14; \quad \sqrt{x^2 + y^2} = S = x
\]

The flow net is plotted in Fig. 77-1

\[
q = kS = \left(\frac{2 \times 10^{-5} \text{ mm/sec}}{1000}\right) 5.14 \text{ m}
\]

\[
q = 1.03 \times 10^{-7} \text{ (m}^3/\text{sec})/\text{m}
\]

Alternate method  From Eq. 70-1,

\[
q = kh_L \frac{M}{N} = \left(\frac{2 \times 10^{-5}}{1000}\right) (15) \left(\frac{7}{19}\right)
\]

\[
q = 1.1 \times 10^{-7} \text{ (m}^3/\text{sec})/\text{m}
\]
The example problem of Fig. 77-2 illustrates two methods to compute the flow rate through an earth dam. The basic parabola may be a poor estimate of the flow line for some geometries ($L << H$), judgment should be exercised in its use.

Flow Net for Earth Dam on Impervious Foundation

In the homogeneous earth dam shown in Fig. 78-1a, the intersection of the downstream face and the impervious foundation is taken as the focus of the parabola for the top flow line. For slope angles greater than 30°, the chart in Fig. 78-1b may be used to find the position of exit of the top flow line from the downstream face of the dam. This point is somewhat off the basic parabola forming the top flow line, and the flow line may be corrected to include this point. The corrected top flow line exits the downstream face along a direction tangent to the face, because the particles of water as they exit the dam tend to flow in the direction as close to vertical as the configuration of the dam allows. The equation $q = kS$ may be used to find the flow through the net. The example problem of Fig. 79-1 illustrates the method for locating the position of the top flow line. For

---

**Figure 78-1**

(a) Discharge face for earth dam on impervious foundation

(b) Chart for locating exit point of top flow line
homogeneous dams in which $\beta$ is less than 30°, Casagrande (1937) suggests the following equation for the distance $a$ to the intersection of the top flow line.

$$a = \frac{L}{\cos \beta} - \sqrt{\frac{L^2}{\cos^2 \beta} - \frac{H^2}{\sin^2 \beta}}.$$

For $\beta < 30^\circ$, the quantity of flow may be found by

$$q = ka \sin \beta \tan \beta$$

**SEEPAGE FORCES**

Water flowing past a soil particle exerts a drag force on the particle in the direction of flow. The drag force is caused by pressure gradient and by viscous drag. To develop mathematical expressions for the drag forces, consider the free bodies in Fig. 80-1. If the height $H$ of the water surface in the tube is raised, the pressure at the bottom of the soil sample is increased and the drag force on the soil particle becomes greater. When the height $H$ reaches a certain critical value $H_c$, the drag forces and the buoyant weight of the particle are in balance and the soil particles will be washed out of the container. At this critical condition, the pressure force acting on the bottom of the soil sample will just equal the weight of the soil and water mass in the container. The hydraulic gradient existing at this time is called the critical gradient ($i_c$). This condition also occurs at the time at
Figure 80-1 For the one-dimensional vertical flow condition shown in Fig. 80-1a, determine expressions for the critical hydraulic gradient and for the critical seepage force.

(a) Flow illustrating seepage forces

(b) Free-body of saturated soil

(c) Equilibrium forces

The determination of critical gradient from Fig. 80-1b is:

\[ W = \left( \frac{G + e}{1 + e} \right) \gamma_w AL = uA = (H_c + L) \gamma_w A \]

from which

\[ i_c = \frac{H_c}{L} = \frac{G - 1}{1 + e} \]

\( i_c \) is called the critical gradient.

The preceding result may be used to find the seepage force causing a critical condition. From Fig. 80-1c,

\[ W_d = (F_s)_c + B \]
For the whole sample this becomes

\[
(F_s)_c = \frac{G\gamma_wAL}{1 + e} - \gamma_wV_s = \frac{G\gamma_wAL}{1 + e} - \gamma_w(1 - n)AL
\]

\[
= \left( \frac{G\gamma_w}{1 + e} - \frac{\gamma_w}{1 + e} \right) AL = i_c(\gamma_w)V
\]

which individual soil particles are freely suspended in the flowing water and do not rest on adjacent particles. This equilibrium of forces is shown in Fig. 80-1c. From Fig. 80-1b, if \( H \) is less than \( H_c \), the seepage force is proportionately less than \( i_c\gamma_wV \) and is always equal to \( i\gamma_wV \) in the direction of flow. The critical condition described above is called the quick condition, or boiling, and is responsible for the phenomenon called quicksand. It occurs in localities in which upward flow is occurring in fine sandy soils. The critical gradient is sufficient to cause boiling in sandy soils but not in cohesive soils in which the additional strength of the soil due to cohesion must be overcome before soil particles will be washed out of the soil mass.

**PIPING AND BOILING**

Erosion of soil particles along the contact surface between the soil foundation and concrete dams is responsible for many of the failures associated with concrete dams. Boiling and soil heave have been observed at the downstream point where flow emerges from the pervious foundation. This occurs when the exit hydraulic gradient approaches the critical hydraulic gradient. For the special case shown in Fig. 82-1, Terzaghi and Peck (1967) suggested analyzing the critical volume \( ABCD \), where \( CD \) is one-half the depth \( AD \). The average vertical hydraulic gradient over the length \( AD \) is

\[
i_{av} = \frac{(h_A + h_B)/2 - (h_C + h_D)/2}{AD}
\]

where \( h \) is the total head at the indicated points. If this gradient is equal to \( i_c \) in a granular soil, piping is imminent. The factor of safety against piping is

\[
F = \frac{i_c}{i_{av}}
\]

and is normally required to be three or greater. If used in cohesive soil, this method is conservative because of the added cohesive strength.
Lane (1935) investigated the problem of piping by using a different approach. He studied a large selection of concrete dams, some of which had failed due to piping, and compared distances of flow along the soil concrete contact surface. He found that as this distance increased, the piping failures occurred less frequently; he therefore devised the following criteria for design of these dams.

\[
WCR = \frac{WCD}{h_L} = \frac{d_v + d_h/3}{h_L}
\]

in which
- **WCR** = weighted creep ratio
- **WCD** = weighted creep distance
  - \(d_v\) = the vertical distance along the contact path
  - \(d_h\) = the horizontal distance along the contact path
- \(h_L\) = total head loss

The dam is considered safe with respect to erosion if \(WCR > (WCR)_{cr}\.

See Fig. 83-1.

For concrete dams with upstream and downstream cutoff walls, such as shown in Fig. 82-1, the path of least resistance for water flow may be directly
Figure 83-1  Critical weighted creep ratios (WCR)cr.

<table>
<thead>
<tr>
<th>Material</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very fine sand or silt to fine sand</td>
<td>8.5–7</td>
</tr>
<tr>
<td>Medium to coarse sand</td>
<td>6–5</td>
</tr>
<tr>
<td>Fine to coarse gravel</td>
<td>4–3</td>
</tr>
<tr>
<td>Boulders with some cobbles and gravel</td>
<td>2.5</td>
</tr>
<tr>
<td>Soft to medium clay</td>
<td>3–2</td>
</tr>
<tr>
<td>Hard to very hard clay</td>
<td>1.8–1.6</td>
</tr>
</tbody>
</table>

Figure 83-2

Example Problem  From Fig. 82-1, the soil is a silty sand, \( k = 1 \times 10^{-3} \) mm/sec, \( G = 2.65 \), and \( e = 0.60 \). Check the design for safety against downstream erosion by the gradient method and the weighted creep method.

Solution  Gradient method. The water depth is 5.5 m. For an elevation datum at the ground surface,

\[
h_c = h_D = 0, \quad h_A = 1.38, \quad h_B = 0.71
\]

\[
i_{av} = \frac{1.38 + 0.71 - 0 - 0}{2(15 \text{ m})} = 0.070
\]

\[
i_c = \frac{G - 1}{1 + e} = \frac{2.65 - 1}{1 + 0.60} = 1.03
\]

\[
\frac{i_c}{i_{av}} = 14.7 = F
\]

Weighted creep method

\[
(WCD)_{EFGHAD} = 18.7 + 15.3 + \frac{29.0}{3} + 12.2 + 15.3 = 71.2
\]

\[
(WCD)_{EFAD} = 18.7 + 2(29.4) + 15.3 = 92.8
\]

\[
WCR = \frac{71.2}{5.5} = 12.95 > 8 \text{ is safe}
\]
through the soil from $F$ to $A$. The two alternatives for weighted creep distance to be investigated in this case would, therefore, be

1. \[ WCD_1 = \frac{EF + FG + GH + HA + AD}{3}. \]

2. \[ WCD_2 = \frac{EF + 2 FA + AD}. \]

The least value is used in finding the weighted creep ratio. Lane recommended that the coefficient of the length $FA$ in the equation for weighted creep distance have a value of 2, which indicates that he felt the resistance to flow along this line is twice as great as that along contact flow paths in the vertical direction. For this method, distances are considered to be vertical or horizontal if they are within 45° of that direction. Although the value of the factor of safety in Lane's method is not known precisely, the method itself is considered to be conservative. The example problem of Fig. 83-2 illustrates both the gradient method and the weighted creep method to check for safety against piping.

**FILTER DESIGN**

Internal and underdrain systems are used to control pore pressure buildup for a number of applications. One example is the blanket drain in the homogeneous earth dam of Fig. 77-1a. Drains must be designed to meet two basic performance criteria.

- The gradation of the drain material must be such that the fines of the adjacent soil will not migrate through the drain.
- The flow capacity of the drain must be high enough to convey all the seepage water without creating an excess hydrostatic head.

Considerable experimentation (U.S. Bureau of Reclamation) has shown that migration of particles from a fine soil into a coarse soil can be prevented if 15% ($D_{85}$) of the particles of the fine soil are larger than the effective pore size of the coarse soil. The effective pore size of the coarse soil corresponds to about one-fifth the $D_{15}$ of the coarse soil. Criteria to prevent particle migration of the fine soil into the coarse soil can, therefore, be stated as

\[ D_{85} \text{ (fine soil)} \geq \frac{1}{5} D_{15} \text{ (coarse soil)} \]

To convey water effectively, a drain must be considerably more permeable than the soil being drained. For uniform soils, the permeability is roughly proportional to the square of the effective grain-size diameter. Thus, for the drain material (coarse soil) to be twenty-five times (rule-of-thumb limit) as permeable as the material being drained (fine soil), the effective diameter of the coarse
material must be five times the effective diameter of the fine material. This type of reasoning leads to the second design criteria.

\[ D_{15} \text{ (coarse soil)} \geq 5 D_{15} \text{ (fine soil)} \]

In addition to these criteria, the U.S. Bureau of Reclamation recommends that the grain-size distribution curves of the fine and coarse soils be roughly parallel.

Perforated drain pipes are frequently used in various internal and under-drain systems. Migration of fine material into the pipe can occur if the size of the perforations in the pipe are too large relative to the \( D_{85} \) of the soil. In order to prevent migration of fines into the pipe, the following criterion is often used.

\[ \frac{D_{85} \text{ (soil)}}{\text{maximum diameter of perforations}} \geq 2 \]

**PROBLEMS**

2-1 Discuss the distribution of water and water pressure in the capillary zone.

2-2 Is water pressure in the adsorbed layer transmitted equally in all directions? Discuss.

2-3 Find the height of rise in a capillary tube with a radius of 0.01 mm.

2-4 Draw a graph of water pressure versus height for the capillary tube in problem 2-3.

2-5 Discuss natural limitations on the height of rise of capillary water in fine-grained soils.

2-6 Find the pressure head, elevation head, and total head for points \( a \) through \( e \) in each of the following situations.
2-7 For the figure in problem 2-6a, the void ratio is 0.82, \( A \) (cross-sectional area) = 1 m\(^2\), and \( Q = 0.5 \) liter in 15 min.
Find:
(a) \( v \) (velocity) in meters per second.
(b) \( v_a \) (actual velocity).
(c) \( k \) (coefficient of permeability) in meters per second.

2-8 If Fig. 53-1 represents a constant head permeameter and if \( k \) (coefficient of permeability) for the soil is 0.001 m/sec, find \( Q \) (flow volume) in 1 hr. \((a, \text{ area of tube} = 300 \text{ mm}^2; A, \text{ area of sample} = 0.1 \text{ m}^2)\).

2-9 Let Fig. 53-1 represent a falling head permeameter. Use the data in problem 2-8. At a time when the head loss in the soil is 4 m, find \( q \) (rate of flow) in cubic meters per second.

2-10 The water depth in an auger hole (diameter = 0.15 m), which is drilled to a depth 3 m below the water table, rises from 0.2 m to 1.2 m (measured from the bottom of the hole) in 6 min. Estimate the coefficient of permeability of the soil. Use Hooghoudt and Ernst equations.

2-11 Plot on an \( x-y \) coordinate system the head distribution for the following equations. Draw constant head lines. Choose physical boundaries that describe a real problem in fluid flow. (Check the solutions to see if they satisfy LaPlace's equation.)

(a) \( h = 4y + 2xy \)
(b) \( h = x^2 + y^2 \)

2-12 Water flows through a saturated silt formation at the rate of 0.1 m\(^3\)/sec. What would be the rate if the head loss were increased 60% and the flow path doubled in length?

2-13 Refer to Fig. 63-2 (scale 1 mm = 1 m) \( k = 1 \times 10^{-5} \) mm/sec at the position of the element. Find:
(a) Pressure head.
(b) Gradient.
(c) Velocity.
(d) Pore pressure.

2-14 Refer to Fig. 67-1. Choose a position on the constant head line labeled 12 and somewhere inside the soil mass. Find:
(a) Pressure head.
(b) Pore pressure.
(c) Gradient.
2-15  This homogeneous earth dam and its foundation are of the same soil down to the impervious layer. Draw the flow net and estimate the rate of flow through the dam.

\[
\begin{align*}
\text{Prob. 2-15}
\end{align*}
\]

2-16  (a) Determine the flow rate \((\text{m}^3/\text{sec/m})\) through this permeable soil.
(b) Determine the pore pressure at \(A\).
(c) Determine the hydraulic gradient at \(A\).
(d) Determine the seepage force per unit volume at \(A\).

\[
\begin{align*}
\text{Prob. 2-16}
\end{align*}
\]
2-17 Steady flow is established in this homogeneous earth dam \((k = 10^{-3} \text{ mm/sec})\). Draw the flow net. Find the seepage rate per meter.

![Prob. 2-17](image)

2-18 Refer to problem 2-15. The permeable drain becomes clogged. Find the point at which the top flow line intersects the downstream surface of the dam. Find the seepage rate per meter.

2-19 Find the magnitude of the seepage force per unit volume at the center of the soil sample in Figs. 53-1 and 66-1b.

2-20 From the flow net shown below, find:

(a) The flow \((q)\) through the net.
(b) The water pressure \((u)\) in the middle of square \(A\).
(c) The actual velocity in the middle of square \(A\).
(d) The seepage force per unit volume (seepage pressure) at \(A\).
(e) The seepage force on the soil in square \(A\).
(f) The factor of safety against piping using the hydraulic gradient.

![Prob. 2-20](image)

Label all assumed dimensions.

2-21 Investigate the safety against downstream erosion for the dam in Fig. 82-1 (scale \(1 \text{ mm} = 2 \text{ m}\)). The soil is a silty sand, \(G = 2.65\), \(e = 0.55\).

2-22 For the dam shown in problem 2-17, \(k_x = 1 \times 10^{-5} \text{ mm/sec}\) and \(k_y = 2 \times 10^{-6} \text{ mm/sec}\). Draw the flow net for the transformed section and find \(q\) (flow rate).